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Are rational expectations really rational?

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This article investigates whether 'rational expectations' are really rational, i.e. whether they maximize agents' utilities in some way. For that purpose we construct a model where each agent may have a whole array of expectations schemes, including the rational expectations one. We find that the usual argument in favor of the 'rationality' of rational expectations is a partial equilibrium one and that, taking into account the full general equilibrium consequences, rational expectations are dominated (in terms of individual utilities) by 'non-rational' expectations.

1. Introduction

The idea of 'rational expectations' is clearly one which has enjoyed a steady success in the economics profession in the last twenty years or so. Though the general idea is always the same, its exact definition may differ a little according to authors: In the seminal contribution by Muth (1961), the (deterministic) anticipated price is taken as equal to the (mathematical) expected value of this price. In his influential paper, Lucas (1972) assumes that agents know the full distribution of future prices conditional on currently available information. Generally, in most rational expectations models the agents are assumed to know the model as well as the model-maker himself, and to make the best prediction (in the probabilistic sense) of the relevant variables conditional on currently available information, so that for example in deterministic models rational expectations are customarily identified with perfect foresight.

Quite strangely for all these years there was little or no questioning of whether 'rational expectations' so defined are rational or not, i.e. whether they maximize agents' utilities in one way or the other. [1] Of course there is always in the back of everybody's mind the 'common wisdom' argument that rational expectations are 'utility maximizing', simply because, other things equal, an agent will reach a higher utility level if he correctly forecasts the variables relevant to him rather than if he incorrectly forecasts them. However, such an 'other things equal' reasoning is at best a partial equilibrium reasoning, whereas such a question must clearly be posed in a general

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* I wish to thank Bruno Jullien and Marie-Odile Yanelle for useful comments on earlier versions of this paper. Of course I remain solely responsible for remaining errors and opinions expressed.
[1] The word 'utility' is used here as a generic term for whichever criterion the agents maximize (this could be for example profits for firms). We prefer to refer explicitly to utility maximization only, as other criteria have less firm choice-theoretic foundations.

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equilibrium framework, notably in view of the massive use of 'rational expectations' models in macroeconomics.

Our purpose in this article is thus to study in a general equilibrium framework the issue of the individual rationality of rational expectations. For that, instead of simply imposing rational expectations as is usually done, we shall construct a model where each agent may have a whole spectrum of expectations schemes, including of course the rational expectations one. In this model agents maximize utility subject to the expectations scheme they are endowed with, and a Walrasian equilibrium obtains. The resulting ex-post utilities will thus be functions of the expectations schemes of all agents. Using the vocabulary of only one of the theory of games one would expect, in view of the 'common wisdom' argument outlined above, that rational expectations are a 'dominant expectations scheme' for each agent. [2] At the very least one would expect rational expectations to be a 'Nash equilibrium'.

Surprisingly we shall see that, because of general equilibrium effects, rational expectations will usually even fail to be a 'Nash equilibrium'. A simple example will further display a case where a non-rational expectations scheme is dominant for all players. This will naturally lead us to question the 'rationality' of 'rational expectations'.

Given the complexity of many rational expectations models, studying in addition a whole array of different expectations schemes might yield overly complex models escaping the intuition. So, in order not to cloud the issue with irrelevant technicalities, we will purposely construct the simplest possible model allowing to study this problem. The model will be deterministic, and each agent will have only one parameter to forecast, 'rational expectations' corresponding to the true future values of this parameter. We shall see that this 'rational expectations' value will usually not maximize the utility of individual agents.

2. The model

The model comprises two agents (1 and 2), two goods (a and b) and two periods. Variables in the second period will be denoted by a superscript prime $'$ (for the true values) or e for the expected values.

Agent 1 has endowments of good a only, denoted (ω_a, ω'_a), and a utility function $U_1(x_{1a}, x_{1b}, x'_{1a})$ where x_{1a} is his current consumption of good a, x_{1b} his current consumption of good b, x'_{1a} his future consumption of good a. Symmetrically agent 2 has endowments of good b (ω_b, ω'_b) and a utility function denoted $U_2(x_{2a}, x_{2b}, x'_{2b})$.

There is only a single market held in period 1, where agents 1 and 2 can exchange good a against good b at numéraire prices p_a and p_b. In the second period they simply consume their endowment, i.e.:

$$x'_{1a} = \omega'_a, \qquad x'_{2b} = \omega'_b. \tag{1}$$

Expectations schemes come in the picture in the following way: In the first period the agents must form expectations on what their future endowments will be, and they forecast respectively ω^e_a

[2] To be more precise, call ω^e_i agent i's expectations scheme. This scheme is called 'dominant' if it maximizes agent i's utility whatever the other agents' expectations schemes ω^e_j, $j \neq i$. A 'Nash equilibrium' is similarly defined. Of course, in spite of this game theoretic vocabulary, there is no game actually played, as we are only comparing the outcomes of various expectations schemes. The game theory terminology, however, is much more precise here than common language, which is why we use it.

for ω'_a and ω^e_b for ω'_b. The supplies and demands of agents 1 and 2 will thus be conditional on these expectations. For example, the program P_1 giving the demand and supply of agent 1 is,

$$\max \quad U_1(x_{1a}, x_{1b}, x'_{1a}),$$
$$\text{s.t.} \quad \begin{cases} p_a x_{1a} + p_b x_{1b} = p_a \omega_a, \\ x'_{1a} = \omega^e_a, \end{cases} \tag{P_1}$$

which yields x_{1a} and x_{1b} as functions of ω_a, ω^e_a and p_b/p_a. Calling $\phi_a = \omega_a - x_{1a}$ the net supply of good a, we have

$$x_{1a} = \omega_a - \phi_a(\omega_a, \omega^e_a, p_b/p_a), \quad x_{1b} = \frac{p_a}{p_b}\phi_a(\omega_a, \omega^e_a, p_b/p_a). \tag{2}$$

Symmetrically the optimal trades of agent 2 are given by

$$x_{2b} = \omega_b - \phi_b(\omega_b, \omega^e_b, p_b/p_a), \quad x_{2a} = \frac{p_b}{p_a}\phi_b(\omega_b, \omega^e_b, p_b/p_a). \tag{3}$$

The Walrasian equilibrium condition for the market of good a against good b is given by the two equivalent equations

$$x_{1a} + x_{2a} = \omega_a \quad \text{or} \quad x_{1b} + x_{2b} = \omega_b,$$

which yields

$$p_a\phi_a(\omega_a, \omega^e_a, p_b/p_a) = p_b\phi_b(\omega_b, \omega^e_b, p_b/p_a). \tag{4}$$

The final allocations in the first period are given by (2) and (3), where p_b/p_a is a function of ω_a, ω^e_a, ω_b and ω^e_b, as given implicitly by eq. (4).

3. The question and basic result

The question asked initially can now be rephrased in the terms of our model. 'Expectations schemes' for agents 1 and 2 correspond simply to a set of forecasted values ω^e_a and ω^e_b for the future endowments. 'Rational expectations' thus corresponds to

$$\omega^e_a = \omega'_a, \qquad \omega^e_b = \omega'_b.$$

Using eqs. (1), (2), (3) and (4), and inserting the corresponding allocations into the original utility functions U_1 and U_2, one can compute the ex-post utility of each agent, which we shall denote as $V_1(\omega^e_a, \omega^e_b)$ and $V_2(\omega^e_a, \omega^e_b)$.

As indicated in the introduction, a minimal condition for rational expectations to be individually rational is that the 'rational expectations' pair (ω'_a, ω'_b) be a 'Nash equilibrium' corresponding to the 'payoff functions' V_1 and V_2. To find out whether this is true we shall compute first-order variations dV_1 and dV_2 letting ω^e_a and ω^e_b vary in the neighborhood of (ω'_a, ω'_b). At this point the allocations actually attained satisfy the first-order conditions corresponding to program P_1 (and the corresponding program for agent 2) so that we have

$$\frac{1}{p_a}\frac{\partial U_1}{\partial x_{1a}} = \frac{1}{p_b}\frac{\partial U_1}{\partial x_{1b}} = \lambda_1, \tag{5}$$

$$\frac{1}{p_a}\frac{\partial U_2}{\partial x_{2a}} = \frac{1}{p_b}\frac{\partial U_2}{\partial x_{2b}} = \lambda_2, \tag{6}$$

where λ_1 and λ_2 are the 'marginal utilities of numéraire income' for agents 1 and 2, respectively. Using (5), small variations in agent 1's utility are computed as (remember x'_{1a}, which is equal to ω'_a, does not move)

$$\mathrm{d}V_1 = \lambda_1(p_a\,\mathrm{d}x_{1a} + p_b\,\mathrm{d}x_{1b}). \tag{7}$$

Differentiating eqs. (2) and plugging into (7), a number of terms cancel out and we obtain the simple expression

$$\mathrm{d}V_1 = -\lambda_1 p_a \phi_a\,\mathrm{d}\log(p_b/p_a). \tag{8}$$

Similarly we find for agent 2, using eqs. (3) and (6),

$$\mathrm{d}V_2 = \lambda_2 p_b \phi_b\,\mathrm{d}\log(p_b/p_a). \tag{9}$$

There remains only to compute d log(p_b/p_a) as a function of the variations in ω_a^e and ω_b^e. For that we differentiate logarithmically eq. (4) and find

$$(1+\epsilon_{b3}-\epsilon_{a3})\,\mathrm{d}\log(p_b/p_a) = \epsilon_{a2}\,\mathrm{d}\log\omega_a^e - \epsilon_{b2}\,\mathrm{d}\log\omega_b^e, \tag{10}$$

where ϵ_{a2} is the elasticity of ϕ_a with respect to its second argument, and so on. Formulas (8), (9) and (10) show us that, as soon as either ϵ_{a2} or ϵ_{b2} is different from zero, a deviation from rational expectations will improve the utility of at least one of the agents, and thus the rational expectations point (ω'_a, ω'_b) will not be a 'Nash-equilibrium'.

More specifically if ϵ_{a2} is different from zero, a deviation $\mathrm{d}\omega_a^e$ such that $\epsilon_{a2}\cdot\mathrm{d}\omega_a^e<0$ will increase agent 1's utility, and symmetrically for agent 2. Note that in this model nonzero ϵ_{a2} and ϵ_{b2} simply means that expectations actually enter the demand and supply functions [eqs. (2) and (3)], and, as eq. (4) shows, this is a necessary condition for expectations to matter at all in the final outcome. So, whenever expectations matter, rational expectations do not maximize each individual's utility.

4. An example

Let us take the following utility functions:

$$U_1 = \alpha\log(x_{1a}+x'_{1a}) + (1-\alpha)\log x_{1b},$$

$$U_2 = \beta\log(x_{2b}+x'_{2b}) + (1-\beta)\log x_{2a}.$$

Simple calculations give us the equilibrium relative prices and utilities as functions of ω_a^e and ω_b^e:

$$\frac{p_b}{p_a}=\frac{(1-\alpha)(\omega_a+\omega_a^e)}{(1-\beta)(\omega_b+\omega_b^e)},$$

$$V_1=\alpha\ \log[\alpha\omega_a+\omega_a'-(1-\alpha)\omega_a^e]+(1-\alpha)\ \log[(1-\beta)(\omega_b+\omega_b^e)],$$

$$V_2=\beta\ \log[\beta\omega_b+\omega_b'-(1-\beta)\omega_b^e]+(1-\beta)\ \log[(1-\alpha)(\omega_a+\omega_a^e)].$$

We see immediately that the best forecasts of agent 1 and 2 , from their individual point of view, are respectively $\omega_a^e=0$ and $\omega_b^e=0$ irrespective of the other's forecast, so that these non-rational expectations are actually 'dominant expectations schemes' and rational expectations are never individually rational.

5. Conclusions

The investigation pursued in this paper has led us to a result which may be surprising to a number of people: In a general equilibrium context rational expectations are not individually rational in the usual sense of the word; that is, rational expectations do not maximize individual utilities.

The mechanism at work here, as particularly evident in eqs. (8) and (9), is that the losses incurred in making computations with wrong expectations are outweighed by the benefits due to the changes in the 'terms of trade'. This 'general equilibrium' effect was clearly the element missing in the 'common wisdom' argument in favor of rational expectations.

For the simplicity of exposition we chose the forecasted variable to be each agent's future endowments. Our results thus display some mathematical similarities with those of Hurwicz (1972) or Postlewaite (1979) who, in a different atemporal framework without expectations, showed that it could be individually rational to misrepresent one's true preferences or endowments. In these papers, however, the agents fully knew all their endowments, whereas here agents do *not* know their future endowments, on which they have to make forecasts. Furthermore, the mechanism we displayed in this paper would clearly be also at work in more general settings where not only individual variables, but also market ones, such as future prices, would have to be forecasted. The formalization would be quite heavier, involving in particular many more markets, but it is to be expected that there too rational expectations would not be individually rational.

Of course one may conjecture that if agents are negligible, in the sense that their individual expectations have negligible effects on the terms of trade, then rational expectations should be individually rational, at least in the Walrasian context considered here. One must be aware, however, that this would be anyway only a special limit case, and that in the general case 'rational expectations' do not seem to have an actual claim on rationality.

The consequences should be at least twofold. First, though 'rational expectations' are obviously a very important benchmark case, economists should work more on other expectations schemes (and notably everything concerned with learning). Secondly, the terminology 'rational expectations' should be abandoned in favor of a perhaps less glamorous, but more accurate terminology than the misleading 'rational expectations' one. The denomination 'perfect expectations', which would extend to all models, stochastic or not, the usual 'perfect foresight' terminology of deterministic models, might do... perfectly. [3]

[3] As another example Lindbeck (1989) proposes the quite accurate terminology of 'model-consistent expectations'.

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