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On the optimality of activist policies with a less informed government☆

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Abstract

I investigate whether a government should lead an activist policy in a rigorous utility maximizing framework under rational expectations. The economy is one with preset wages, and is subject to both demand and supply shocks. It is assumed that the government can never act on the basis of information superior to that of the private sector. Moreover wages are set after government policy has been carried out. We find that the optimal policy is nevertheless an activist countercyclical one. It has the remarkable property that, although the economy is hit each period by stochastic shocks *after* wages have been preset, this optimal policy will nevertheless succeed in keeping the economy on a full employment track. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The purpose of this article is to reexamine in a rigorous utility maximizing framework with rational expectations the traditional debate about the desirability

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of an 'activist' governmental policy when nominal rigidities are present. We shall construct for that purpose a simple model of an economy with preset wages. This economy is subject to both demand and supply shocks. We shall find out that, eventhough the government is assumed to have never more information than the private sector, his optimal policy will be an activist one.

The reason why this study is cast within the framework of an economy with preset wages is that the existence of rigid wages (or prices) was at the heart of the traditional 'Keynesian' case for activist stabilization policies: in such a situation negative demand shocks lead to inefficient underemployment of resources, which, it was believed, the government should alleviate through countercyclical monetary or fiscal policies.

An almost mortal blow, however, was brought to this view by the contributions of Lucas (1972, 1976) and Sargent and Wallace (1975, 1976). In particular Sargent and Wallace (1975) showed that, in a class of models which included the models with preset prices and wages studied at the time, the scope for output and employment stabilization disappears if the private sector has rational expectations and is allowed to react to the same informations as the government. This critique is a compelling one since, even if the government has more information than the private sector, it could be considered as one of its duties to release this superior information to private agents, and intervene only if this was not sufficient.[1]

Subsequently the important idea that a less informed government can nevertheless have stabilizing powers was developed in insightful articles by Turnovsky (1980), Weiss (1980, 1982), King (1982, 1983) and Andersen (1986). All these papers imbed a sophisticated treatment of rational expectations into an otherwise fairly traditional framework, with a priori given demand–supply functions and government objectives. Of course the question naturally arises, as for all models with no explicit microfoundations, of whether these important results will carry over in a model where demands, supplies and the objectives of the government all derive from explicit maximization.

So the purpose of this article is to reexamine this issue in a rigorous maximizing model with preset wages, where the government is less informed and more constrained than the private sector. More precisely we shall notably assume that: (i) the government takes his policy actions on the basis of information which is never superior to that of the private sector, (ii) the private sector sets wages after the government has decided on policy for the same period, so that

[1] We may note that many famous contributions favourable to activism are explicitly based on the assumption that the government can take action on the basis of superior information, and are therefore vulnerable to the Sargent–Wallace critique. For example Fischer (1977) himself shows that, in his model, the scope for stabilization disappears if the private sector is allowed to react to the same informations as the government.

the government cannot 'surprise' the private sector while the latter is locked into fixed wages agreements.

Inspite of these restrictions, we shall show that the optimal policy is nevertheless an activist one. We shall notably obtain the remarkable result that, although the economy is hit in each period by stochastic demand and supply shocks *after* wages have been preset, our optimal policy will nevertheless succeed in keeping the economy on a full employment track.

2. The model

2.1. An OLG model

We shall consider a monetary overlapping generations model (Samuelson, 1958) with production. At this stage some readers may wonder why we use an OLG model and not, say, a model of an infinitely lived representative agent with money (whether with a cash in advance constraint or money in the utility function), a model which has become quite popular with the advent of RBC models. There are actually several reasons for this choice.

The first reason is obvious: because of the simple OLG structure, we shall be able to obtain closed-form solutions throughout, which makes the solution of the problem and the economic interpretations crystal clear. This would not have been the case with a more complex structure, where no exact analytical solution could have been found.

A second reason is that we shall be using a policy of transfers, which is essentially a fiscal policy. But in the infinitely lived agent model, Ricardian equivalence holds, which means that fiscal policy in a particular period does not have any effect per se. So in order to carry out our investigation we would have had to add some additional imperfections, and there could always be the suspicion that the results are actually due to this extra imperfection.

Third, had we studied the matter in a model of an infinitely lived agent with money, we would have encountered Friedman's (1969) prescription of setting the nominal interest rate to zero. But under such policy Sargent and Wallace (1975) have shown that Walrasian equilibrium prices and wages are potentially indeterminate, clearly a bad start for a model where wages are equal to expected Walrasian wages. The OLG model escapes that problem (for more formal developments on this issue, see notably Bénassy, 2000).

Of course, as compared with the infinite life model with money, our model does not distinguish between money and other financial stores of value. This is actually not a problem since we study a fiscal type policy (and not, say, an 'open-market type' monetary policy), and this is thus a very minor drawback compared to all the above advantages.

2.2. The agents

The economy includes representative firms and households, and the government.

Households of generation t live for two periods. They work L_t and consume C_t in period t, consume C'_{t+1} in period $t+1$. They save under the form of money, which is the sole asset in the economy,[2] and maximize the expected value of their utility U_t, with

$$U_t = \alpha_t \operatorname{Log} C_t + \operatorname{Log} C'_{t+1} - (1+\alpha_t)L_t \tag{1}$$

where the α_t's are positive i.i.d. stochastic variables whose variations represent demand shocks (the propensity to consume in period t is, as we shall see below, $\alpha_t/1+\alpha_t$). The coefficient $1+\alpha_t$ in the disutility of labor has been chosen so as to yield a constant Walrasian labor supply in the absence of government intervention (Section 3 below). In that way, variations in α_t have the characteristics of a 'pure demand shock'.

The representative firm in period t has a production function:

$$Y_t = Z_t L_t \tag{2}$$

where Y_t is output, L_t labor input and Z_t a technology shock common to all firms. We assume that the firms belong to the young households, to which they distribute their profits, if any.

Government has one policy instrument: It can increase or decrease the money stock through fiscal transfers to the old household. As we shall see next, we shall constrain government transfers to be conditional only on variables already known to the private sector.

2.3. The timing of events

As in all such models, the timing of actions and information is important, so we shall now spell things precisely.

Old households enter period t holding a quantity of money M_{t-1} carried from the previous period. The government gives them in a first step a lump sum transfer τ_t, so that the old households are now endowed with a quantity of money M_t:

$$M_t = M_{t-1} + \tau_t. \tag{3}$$

[2] Since there is a unique financial asset, what we shall call money, and denote as M_t, must be understood, as usual in OLG models, as the sum of net financial assets. Although we shall keep the convenient denomination of 'money' for short, that economic interpretation should be kept in mind throughout.

Call I_t the information set in period t, which includes the values of all observable macroeconomic variables up to t included. In order to reflect the fact that government policy in period t can only react to past developments, we shall assume that the government's policy variable τ_t is a function[3] only of variables belonging to I_{t-1}, which the private sector already knows.

In a second step the wage is set by the private sector at its expected market clearing value, without knowing the values of period t shocks α_t and Z_t.

Finally the shocks become public knowledge and transactions are carried out.

We may note that, as indicated in the introduction, the government does not have the opportunity to use policy to 'surprise' the private sector while he is locked into binding nominal contracts, since the contracts are signed *after* the government has made its transfer. Also the transfer in period t is based on information up to $t-1$, so that the government is no more informed than private agents.

3. Walrasian equilibrium

In order to contrast the results with the preset wage economy, we shall study first the Walrasian equilibria of this economy.

Call P_t and W_t the price and nominal wage. The real wage is equal to the marginal productivity of labor:

$$W_t/P_t = Z_t. \tag{4}$$

Now let us write the maximization program of the young household born in t. He receives profits $\Pi_t = P_t Y_t - W_t L_t$ when young, and a transfer τ_{t+1} from the government when old. He saves a quantity of money m_t. So his program is

$$\text{Maximize} \quad \mathrm{E}_t[\alpha_t \operatorname{Log} C_t + \operatorname{Log} C'_{t+1} - (1+\alpha_t)L_t] \quad \text{s.t.}$$

$$P_t C_t + m_t = W_t L_t + \Pi_t,$$

$$P_{t+1} C'_{t+1} = m_t + \tau_{t+1}.$$

Note that, since τ_{t+1} is a function of variables up to period t, it is known to the household when deciding on quantities supplied and demanded. The first order conditions for this program yield

$$P_t C_t = \frac{\alpha_t}{1+\alpha_t}(W_t L_t + \Pi_t + \tau_{t+1}) = \frac{\alpha_t}{1+\alpha_t}(P_t Y_t + \tau_{t+1}), \tag{5}$$

$$L_t^s = \frac{W_t - \Pi_t - \tau_{t+1}}{W_t}. \tag{6}$$

[3] We shall thus consider only deterministic policy rules in this paper. As we shall see below, deterministic policies are sufficient to reach optimal allocations, and stochastic policies would only add unwanted noise. Although this will not be the case for the optimal policy found below, these policies could be time dependent.

Eq. (5) is the usual consumption function, while Eq. (6) gives the Walrasian supply of labor.

The equilibrium condition on the goods market is

$$C_t + C'_t = Y_t = Z_t L_t \tag{7}$$

where C'_t, consumption demand by old consumers, is simply

$$C'_t = M_t / P_t. \tag{8}$$

Eqs. (4)–(8) determine all equilibrium values, which depend on M_t and $M_{t+1} = M_t + \tau_{t+1}$. They are computed as

$$C_t = \frac{\alpha_t Z_t}{1 + \alpha_t}, \quad C'_t = \frac{Z_t M_t}{(1 + \alpha_t) M_{t+1}}, \tag{9}$$

$$L_t = \frac{1}{1 + \alpha_t}\left(\alpha_t + \frac{M_t}{M_{t+1}}\right), \tag{10}$$

$$W^*_t = (1 + \alpha_t) M_{t+1}, \quad P^*_t = \frac{(1 + \alpha_t) M_{t+1}}{Z_t}. \tag{11}$$

We see from Eq. (10) that, as we indicated in Section 2 above, if there are no transfers, i.e. if $M_{t+1} = M_t$, then the Walrasian quantity of labor is constant and equal to one.

4. Optimality

4.1. The criterion

In order to assess the optimality properties of various government policies, both in the Walrasian and the non-Walrasian case, we need to have a criterion. Clearly with an infinity of generations the Pareto optimality criterion would not be demanding enough. We shall thus use the criterion proposed by Samuelson for the overlapping generations model (Samuelson, 1967, 1968; Abel 1987) and assume that in period t the government maximizes the function V_t, with

$$V_t = \mathrm{E}_t \sum_{s=t-1}^{\infty} \beta^{s-t} U_s. \tag{12}$$

Note that the sum starts at $s = t - 1$ because the household born in $t - 1$ is still alive in t. The limit case $\beta = 1$ corresponds to maximizing the representative household's expected utility.

Rearranging a little the terms in the infinite sum (12), we find that, up to a constant, the criterion V_t can be rewritten under the more convenient form:

$$V_t = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \Delta_s, \tag{13}$$

$$\Delta_t = \alpha_t \operatorname{Log} C_t + \frac{\operatorname{Log} C'_t}{\beta} - (1+\alpha_t) L_t. \tag{14}$$

4.2. Optimal policy in the Walrasian case

Let us begin our investigation of optimal policies with the Walrasian case. In order to find the best policy, we simply insert the equilibrium values found in (9) and (10) into the criterion (13)–(14). The term corresponding to period t is equal to

$$\Delta_t = \alpha_t \operatorname{Log}\left[\frac{\alpha_t Z_t}{1+\alpha_t}\right] + \frac{1}{\beta} \operatorname{Log}\left[\frac{Z_t M_t}{(1+\alpha_t) M_{t+1}}\right] - \left(\alpha_t + \frac{M_t}{M_{t+1}}\right). \tag{15}$$

Maximizing this with respect to M_{t+1}/M_t, we immediately find the optimal policy under the Walrasian regime

$$M_{t+1}/M_t = \beta. \tag{16}$$

Looking at this optimal policy we may note two things:

- The first is that optimal policy (16) is identical to that found, following Friedman's (1969) famous 'optimal quantity of money' article, by numerous authors[4] working with infinitely lived representative agents with a discount rate β.
- Secondly this optimal policy is a typical non-activist one, since the policy defined by (16) does not depend on any event, past or present.

We shall now see that the introduction of preset wages changes things quite drastically.

5. Preset wages

We shall now assume that firms and workers sign wage contracts at the beginning of period t, based on information available then (which does *not*

[4] See Dornbusch and Frenkel (1973), Grandmont and Younès (1973), Brock (1975), and many others since.

include the values of α_t and Z_t) and that at this wage households will supply the quantity of labor demanded by firms. It will be assumed here, in order not to add any further distorsion, that the preset wage is equal to the expected value of the Walrasian wage, i.e. using formula (11):

$$W_t = \mathrm{E}_{t-1} W_t^* = \mathrm{E}_{t-1}[(1+\alpha_t)M_{t+1}]. \tag{17}$$

5.1. Computing the equilibrium

We may note that all equilibrium equations (4)–(8) still hold, with the only exception of Eq. (6), expressing that the household is on his labor supply curve, which is replaced by Eq. (17). Combining these equations, we find that the preset wage equilibrium is characterized by the following relations:

$$L_t = \frac{\alpha_t M_{t+1} + M_t}{W_t}, \tag{18}$$

$$Y_t = Z_t L_t, \tag{19}$$

$$P_t = W_t / Z_t, \tag{20}$$

$$C_t = \alpha_t M_{t+1} Z_t / W_t, \tag{21}$$

$$C_t' = M_t Z_t / W_t. \tag{22}$$

5.2. The suboptimality of non-activist policies

In order to show the suboptimality of non-activist policies, we shall now study what will happen if the government follows policy (16), which was optimal under Walrasian market clearing. In view of (16) and (17), the preset wage is:

$$W_t = \beta(1+\alpha_a)M_t \tag{23}$$

where $\alpha_a = \mathrm{E}_{t-1}\alpha_t$ (the subscript a meaning average). Eqs. (18)–(22) yield the following values:

$$L_t = \frac{1+\beta\alpha_t}{\beta(1+\alpha_a)}, \tag{24}$$

$$C_t = \frac{\alpha_t Z_t}{1+\alpha_a}, \quad C_t' = \frac{Z_t}{\beta(1+\alpha_a)}. \tag{25}$$

It is easy to check that the allocations defined by (24)–(25) are not even a Pareto optimum. Looking now at the labor market, we see, combining (6), (16) and (23), that the supply of labor is equal to:

$$L_t^s = \frac{1+\beta\alpha_a}{\beta(1+\alpha_a)}. \tag{26}$$

Comparing (24) and (26), we see that the economy will display either unemployment (when $\alpha_t < \alpha_a$) or overemployment (when $\alpha_t > \alpha_a$), both creating inefficiencies. We shall now show that an activist policy allows to do much better.

6. The optimality of activist policies

Finding an optimal policy consists in finding a strategy where: (i) τ_t, or M_t, are function only of variables in I_{t-1}; (ii) the resulting equilibrium values maximize the utility function V_t in (12)–(14) for this class of policies.

In order to find the optimal policy in a simple manner we insert into the criterion (14) the 'fixwage equilibrium' values of C_t, C'_t, and L_t found above (Eqs. (18)–(22)). In period t the government will thus maximize the expected value of the following quantity:

$$\alpha_t \operatorname{Log}\left[\frac{\alpha_t M_{t+1} Z_t}{W_t}\right] + \frac{1}{\beta}\operatorname{Log}\left[\frac{M_t Z_t}{W_t}\right] - (1+\alpha_t)\left[\frac{\alpha_t M_{t+1}}{W_t} + \frac{M_t}{W_t}\right] \tag{27}$$

subject to (17). In this maximization M_t is inherited from the previous period, M_{t+1} can be chosen conditional on the value of all shocks, while W_t is predetermined according to Eq. (17). As it turns out, constraint (17) is actually not binding at the optimum, and consequently one obtains exactly the same solution maximizing the expected value of (27) with respect to M_{t+1} and W_t. Because the whole problem is homogeneous of degree zero in M_{t+1}, W_t and M_t, we shall actually maximize with respect to the ratios M_{t+1}/W_t and W_t/M_t. Let us first maximize (27) in M_{t+1}/W_t. This yields

$$M_{t+1} = W_t/(1+\alpha_t). \tag{28}$$

We may immediately note that, by combining (11) and (28), we obtain

$$W_t^* = (1+\alpha_t)M_{t+1} = W_t. \tag{29}$$

Under policy rule (28) the Walrasian wage in period t is *independent* of period t shocks, α_t and Z_t, and thus fully predetermined. As a result the contract wage W_t, which is equal to the expected value of W_t^*, is always at its market clearing value, and therefore under policy (28) the economy will always be at full employment!

Now inserting the value of M_{t+1}/W_t so obtained into the expression of the expected value of Δ_t, we obtain, up to a constant term

$$\frac{1}{\beta}\operatorname{Log}\left(\frac{M_t}{W_t}\right) - \frac{(1+\alpha_a)M_t}{W_t}.$$

Maximization of this term in W_t/M_t yields

$$W_t = \beta(1 + \alpha_a)M_t. \tag{30}$$

Combining (28) and (30), we finally obtain the formula for the optimal monetary policy:

$$M_{t+1}/M_t = \beta(1 + \alpha_a)/(1 + \alpha_t). \tag{31}$$

We see that rule (31) combines in a nutshell both some Friedmanian and Keynesian insights. Indeed we can note first that, if there were no demand shocks, i.e. if α_t was constant, Eq. (31) would yield $M_{t+1} = \beta M_t$, the traditional 'Friedmanian' rule (16), which we found to be optimal in the Walrasian case. However we see also that, as soon as demand shocks are present, optimal policy will call for the government to respond countercyclically to these shocks, since a negative demand shock today (low α_t) will trigger large transfers tomorrow (high τ_{t+1} and M_{t+1}) and conversely for a positive demand shock. Optimal policy is thus an activist one.

7. Conclusions

We constructed in this article a model of an economy with preset wages where agents maximize under rational expectations, and showed that it was optimal for a 'less informed' government to lead nevertheless an activist countercyclical fiscal policy. With our specification of the model the government can even maintain the economy at all times on a full employment trajectory.

The fact that a government with no more information than the private sector can nevertheless succeed in stabilizing the economy, eventhough wages are set in advance without knowledge of the shocks, may be somewhat surprising, so we shall give here a quick intuition for the cause of that remarkable result. Let us rewrite the household's consumption function (5):

$$P_t C_t = \frac{\alpha_t}{1 + \alpha_t}(P_t Y_t + \tau_{t+1}).$$

Now consider the situation where the wage has been already set and assume that a negative demand shock (a low α_t) hits the economy. If the government led no systematic policy this shock would clearly lead, in view of the above consumption function, to a *decrease* in the demand for goods and labor, and therefore to an underemployment of labor. But if the government is known to lead the countercyclical policy (31), then the private sector will know in advance that the future transfer τ_{t+1} will be high, and from the above formula this will tend in the contrary to *increase* the demand for goods and labor. When the policy is calibrated to be (31), these two conflicting effects cancel out, and the economy remains at full employment. Of course the zero unemployment result is due to our particular specifications, but the optimality of an activist policy is a robust result, as the appendices show.

An issue often raised against traditional activist policies is that they might impart an inflationary bias to the economy. So we should note that this is not at all the case with the optimal policy in this paper, since all nominal values will increase on average at the rate β, thus following a nonincreasing trend. The traditional opposition between employment stabilization and price stability therefore does not hold here.

Appendix A

The model we have used in the main text has the particular features that the disutility of labor (Eq. (1)) and the production function (Eq. (2)) are both linear in labor. We shall now develop a simple argument to show that our result on the optimality of activist policies has nothing to do with these linearities, and continues to hold under nonlinear specifications. For that purpose we shall use the more general nonlinear utility and production functions:

$$U_t = \alpha_t \operatorname{Log} C_t + \operatorname{Log} C'_{t+1} - (1 + \alpha_t)L_t^\theta \qquad \theta > 1, \tag{A.1}$$

$$Y_t = Z_t L_t^v, \qquad 0 < v \le 1. \tag{A.2}$$

All of the arguments that have been developed in the main text could be developed similarly, with the important difference that we do not have an explicit solution to this more general problem. What we know is that, at the end, there is an optimal policy function of the government which will be of the form:

$$M_{t+1}/M_t = \Phi(\alpha_t, Z_t, \theta, v). \tag{A.3}$$

Since we conducted our investigation in the main text with the values $\theta = 1$ and $v = 1$, we know further that

$$\Phi(\alpha_t, Z_t, 1, 1) = \beta(1 + \alpha_a)/(1 + \alpha_t). \tag{A.4}$$

Because the problem is continuous, we know that for θ and v close to 1, the optimal response function will be close to $\beta(1 + \alpha_a)/(1 + \alpha_t)$, which means that governmental policy will respond 'actively' to demand shocks. This shows that our result on the optimality of activist policies is actually robust, and does not depend on the linearity of some specifications.

Appendix B

The previous appendix showed us that our results on the optimality of activist policies continue to hold if we assume nearly linear production function and disutility of labor. We will show now that our results continue to hold with a production function that can be quite far from linear. For that purpose we shall use a Cobb–Douglas production function:

$$Y_t = Z_t F(L_t) = Z_t L_t^{\nu} \tag{B.1}$$

where $0 < \nu \leq 1$. The household's utility function is the same (Eq. (1)).

B.1. Walrasian equilibrium

All equations concerning the household are the same as in Section 2. The real wage is equal to the marginal productivity of labor:

$$W_t/P_t = Z_t F'(L_t) = Z_t \nu L_t^{\nu - 1}. \tag{B.2}$$

Solving system (5)–(8) and (B.2), we find the following Walrasian equilibrium values:

$$W_t^* = (1 + \alpha_t) M_{t+1}, \quad L_t = \nu \frac{M_t + \alpha_t M_{t+1}}{(1 + \alpha_t) M_{t+1}}, \tag{B.3}$$

$$C_t = \frac{\alpha_t M_{t+1}}{M_t + \alpha_t M_{t+1}} Z_t L_t^{\nu}, \quad C_t' = \frac{M_t}{M_t + \alpha_t M_{t+1}} Z_t L_t^{\nu}. \tag{B.4}$$

B.2. Optimal policy in the Walrasian case

We shall obtain the optimal policy by maximizing in each period the following criterion:

$$\Delta_t = \alpha_t \operatorname{Log} C_t + \frac{\operatorname{Log} C_t'}{\beta} - (1 + \alpha_t) L_t \tag{B.5}$$

where C_t, C_t' and L_t are given by formulas (B.3) and (B.4) above. The solution to this program is

$$M_{t+1} = \beta M_t \tag{B.6}$$

in which we recognize again the 'Friedman rule'.

B.3. Preset wages

We now assume that firms and workers sign wage contracts at the beginning of period t:

$$W_t = \mathrm{E}_{t-1} W_t^* = \mathrm{E}_{t-1}[(1 + \alpha_t) M_{t+1}]. \tag{B.7}$$

Equilibrium equations (5), (7), (8) and (B.2) still hold. Eq. (6) is replaced by Eq. (B.7). Combining these equations, we find that the preset wage equilibrium is characterized by the following relations:

$$L_t = v\frac{M_t + \alpha_t M_{t+1}}{W_t}, \tag{B.8}$$

$$C_t = \frac{\alpha_t M_{t+1}}{M_t + \alpha_t M_{t+1}} Z_t L_t^v, \quad C'_t = \frac{M_t}{M_t + \alpha_t M_{t+1}} Z_t L_t^v. \tag{B.9}$$

B.4. Evaluating a change of policy

We shall now insert the values found for the fixwage equilibrium (Eqs. (B.8), (B.9)) into criterion (B.5). Calling π_t the probabilities attached to each realization of the α_t's:

$$\Omega_t = \mathrm{E}_t(\Delta_t) = \sum \pi_t \left\{ \alpha_t \operatorname{Log}\left[\frac{\alpha_t M_{t+1}}{M_t + \alpha_t M_{t+1}} Z_t L_t^v \right] \right\}$$

$$+ \sum \pi_t \left\{ \frac{1}{\beta} \operatorname{Log}\left[\frac{M_t}{M_t + \alpha_t M_{t+1}} Z_t L_t^v \right] - (1 + \alpha_t) L_t \right\}. \tag{B.10}$$

Let us now consider potential changes in future policy M_{t+1} and differentiate Ω_t with respect to them. We find after a little manipulation:

$$\mathrm{d}\Omega_t = \sum \pi_t \left\{ \frac{\alpha_t(\beta M_t - M_{t+1})}{\beta M_{t+1}(M_t + \alpha_t M_{t+1})} \right\} \mathrm{d}M_{t+1}$$

$$+ \sum \pi_t \left(\frac{1 + \beta\alpha_t}{\beta} \right) \frac{W_t}{M_t + \alpha_t M_{t+1}} \mathrm{d}L_t - \sum \pi_t (1 + \alpha_t) \mathrm{d}L_t. \tag{B.11}$$

Our proof of the optimality of activist policies will now be achieved in two steps: first we shall show that among all nonactivist policies the 'Friedman rule' $M_{t+1} = \beta M_t$ is the best. Secondly we shall show that this rule is actually dominated by an activist one.

B.5. Non-activist policies

Let us thus consider first non-activist policies of the type:

$$M_{t+1} = \gamma M_t. \tag{B.12}$$

In such a case the preset wage and employment level are equal to

$$W_t = \sum \pi_t (1 + \alpha_t) M_{t+1} = (1 + \alpha_\mathrm{a}) \gamma M_t, \tag{B.13}$$

$$L_t = v\frac{M_t + \alpha_t M_{t+1}}{W_t} = v\frac{1 + \gamma\alpha_t}{\gamma(1 + \alpha_\mathrm{a})}. \tag{B.14}$$

Using (B.11)–(B.14) we can compute

$$\mathrm{d}\Omega_t = \sum \pi_t \left\{ \frac{(v + \gamma\alpha_t)(\beta - \gamma)}{\beta\gamma^2(1 + \gamma\alpha_t)} \right\} \mathrm{d}\gamma. \tag{B.15}$$

We see that Ω_t will be maximum for $\gamma = \beta$, so that the 'Friedman rule' is still the best among all nonactivist policies.

B.6. Activist policies

We shall now start from this optimal nonactivist policy $\gamma = \beta$ and see whether the situation can be improved by leading an activist policy. We can compute (B.11) starting from $M_{t+1} = \beta M_t$, and we find

$$\mathrm{d}\Omega_t = \sum \pi_t (\alpha_\mathrm{a} - \alpha_t)\,\mathrm{d}L_t. \tag{B.16}$$

Let us now consider infinitesimal countercyclical activist policies of the type

$$\mathrm{d}M_{t+1} = \frac{(\alpha_\mathrm{a} - \alpha_t)\,\mathrm{d}\phi}{1 + \alpha_t} \tag{B.17}$$

where $\mathrm{d}\phi$ is a small positive variation. Then

$$\mathrm{d}W_t = \sum \pi_t (1 + \alpha_t)\,\mathrm{d}M_{t+1} = \sum \pi_t (\alpha_\mathrm{a} - \alpha_t)\,\mathrm{d}\phi = 0. \tag{B.18}$$

Now since $L_t = v(M_t + \alpha_t M_{t+1})/W_t$ (Eq. (B.8)):

$$\mathrm{d}L_t = \frac{v\alpha_t\,\mathrm{d}M_{t+1}}{W_t} = \frac{v\alpha_t(\alpha_\mathrm{a} - \alpha_t)\,\mathrm{d}\phi}{(1 + \alpha_t)W_t}, \tag{B.19}$$

$$\mathrm{d}\Omega_t = \sum \pi_t \frac{\alpha_t(\alpha_t - \alpha_\mathrm{a})^2\mathrm{d}\phi}{\beta(1 + \alpha_t)(1 + \alpha_\mathrm{a})M_t} > 0. \tag{B.20}$$

The new activist policy thus dominates the best nonactivist policy.

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