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Review of Economic Dynamics 8 (2005) 651–667

Review of Economic Dynamics

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# Interest rate rules, price determinacy and the value of money in a non-Ricardian world

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Received 19 June 2003; revised 25 February 2004

Available online 23 May 2005

**Abstract**

This article studies under which conditions interest rate rules "à la Taylor" [1993. Discretion versus policy rules in practice. Carnegie–Rochester Conference Series on Public Policy 39, 195–214] lead to price determinacy. We scrutinize notably two famous results, which are standard in the traditional "Ricardian" model with a single dynasty of consumers: (1) a pure interest rate peg leads to nominal price indeterminacy; (2) a strong reaction (usually more than one for one) of nominal interest rates to inflation is conducive to price determinacy (the Taylor principle). This article extends the analysis to rigorous dynamic non-Ricardian models. The results turn out to be quite different, since notably prices may be determinate if the interest rate responds less than one for one to inflation, and even under a pure interest rate peg.


*JEL classification:* E43; E50; E52; E58

*Keywords:* Interest rate rules; Price determinacy; Monetary policy rules; Taylor principle; Interest rate peg; Pigou effect; Real balance effect

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1094-2025/$ – see front matter 
doi:10.1016/j.red.2005.01.013

## 1. Introduction

Following Taylor's (1993) seminal article, there has been recently a very strong renewal of interest in the study of interest rate rules for monetary authorities (for a survey of recent work, see for example McCallum, 1999; Taylor, 1999). In line with the recent trends in macroeconomics, several authors quite naturally investigated interest rate policies in rigorous dynamic general equilibrium models.

Most rigorous studies of optimal interest rate rules in such a maximizing framework have been cast in "Ricardian" economies populated with a single dynasty of consumers.[1] These economies have, however, as far as policy analysis is concerned, a number of particular properties, and it thus seems legitimate, in line with the intuition first developed in Bénassy (2000), to extend the analysis of interest rate rules to non-Ricardian economies where new agents enter in each period, and to see whether this makes a difference or not for the analysis. We shall see that it does.

In this article we shall be particularly concerned with the issue of price determinacy under various monetary rules. We shall notably scrutinize two particularly famous results:

- The first one, which originates with the article by Sargent and Wallace (1975), basically says that, under a pure nominal interest rate peg, there is nominal indeterminacy.[2] This means that, if a sequence of prices is an equilibrium, then any sequence multiple of the first one is also an equilibrium. This is no minor problem since many optimal policy packages include the famous "Friedman rule," according to which the nominal interest rate should be set equal to zero.[3]
- The second one is often referred to as the "Taylor principle."[4] The basic idea is that, in order to make prices determinate the central bank should respond "aggressively" to inflation. If interest rates respond only to inflation, a classic result is that, in order to have determinate prices, nominal interest rates should respond more than one for one to inflation.[5]

[1] So we shall use the terminology "Ricardian" for economies with a single dynasty of consumers, and "non-Ricardian" for economies where new consumers arrive in time. This terminology has its root in the "Ricardian equivalence" result (Barro, 1974), according to which, in an economy with a single dynasty of consumers, the timing of (lump sum) taxes is irrelevant as long as the government intertemporally balances its budget, whereas it matters in a non-Ricardian economy. Note that other factors than demographics can make an economy non-Ricardian.

We should point out that this meaning of "Ricardian" is quite different from a later one which differentiates Ricardian and non-Ricardian *policies*, and which has been adopted by authors working on the "fiscal theory of the price level" (see, for example, Kocherlakota and Phelan, 1999, and Woodford, 2003, for lucid expositions).

[2] For a useful taxonomy of various forms of indeterminacy, see McCallum (1986).

[3] This rule originates in Friedman (1969). The intuition is that, since money costs nothing to produce, its services should be priced at zero.

[4] It should be noted that, although Taylor (1993) recommends a strong response of interest rates to inflation, this is not for the reasons explored in this article. The reasoning (see, for example, Taylor, 1998) is that if the nominal interest rate responds more than one for one to inflation, the real interest rate will respond positively to inflation, which should have a stabilizing influence on the economy.

[5] Early results in this direction on price determinacy and monetary rules are found in Leeper (1991).

We shall see that considering non-Ricardian instead of Ricardian economies dramatically modifies the answers to the two above questions.[6] Notably:

– A pure interest rate peg is fully consistent with price determinacy, provided the interest rate satisfies a very mild condition.
– Prices can be determinate even if the interest rate responds less than one for one to inflation. In fact, from the above result, it needs not even respond at all.

The rest of the article is organized as follows: Section 2 presents the model. Section 3 derives the dynamic equilibrium equations. Section 4 describes, for the sake of comparison, some traditional results in the Ricardian framework. Section 5 derives a sufficient condition for price determinacy under an interest rate peg. Section 6 studies whether the Taylor principle is still relevant in the non-Ricardian model. Section 7 gives a number of economic interpretations. Section 8 extends the results to a framework with a forward looking Phillips curve. Section 9 introduces a more general fiscal policy. Section 10 concludes.

## 2. The model

The model consists of households and the government, itself comprising a fiscal authority (which sets taxes) and a monetary authority (which sets nominal interest rates).

In order to have a non-Ricardian structure, we shall use a model adapted from that of Weil (1987, 1991).[7] New "generations" of households are born each period, but nobody ever dies. Call $N_t$ the number of households alive at time $t$. So $N_t - N_{t-1}$ households are born in period $t$, with $N_t \geqslant N_{t-1}$. We will actually mainly work below with the case where the population grows at the constant rate $n \geqslant 0$, so that $N_t = (1+n)^t$.

### 2.1. Households

Consider a household $j$ (i.e. a household born in period $j$). We denote by $c_{jt}$ and $m_{jt}$ his consumption and money holdings at time $t \geqslant j$. This household receives in periods $t \geqslant j$ an endowment $y_{jt}$ and maximizes in each period $t \geqslant j$ the following utility function:

$$U_{jt} = \sum_{s=t}^{\infty} \beta^{s-t} \operatorname{Log} c_{js}. \tag{1}$$

[6] A few contributions have sought to modify the "traditional" results on interest pegging or the Taylor principle. For example McCallum (1981) advocates linking interest rates to a "nominal anchor" like the price level. Bénassy (2000) introduces the non-Ricardian insight into the issue of interest pegging using a different uniqueness criterion. Benhabib et al. (2001) show the importance of how money enters the utility and production functions. Fair (2003) shows that the Taylor principle does not apply in an econometrically estimated model. Roisland (2003) shows that capital income taxation modifies the Taylor principle.

[7] The difference is that Weil includes money in the utility function, whereas we use a cash in advance constraint. The determinacy conditions are actually the same for the two models.

Household $j$ is submitted in period $t$ to a "cash in advance" constraint:

$$P_t c_{jt} \leqslant m_{jt}. \tag{2}$$

Household $j$ enters period $t$ with a financial wealth $\omega_{jt}$. Transactions occur in two steps. First the bonds market opens, and the household can lend at the nominal interest rate $i_t$ an amount $b_{jt}$ (of course $b_{jt}$ can be negative if the household borrows to obtain liquidity). The rest is kept under the form of money $m_{jt}$, so that:

$$\omega_{jt} = m_{jt} + b_{jt}. \tag{3}$$

Then the goods market opens, and the household sells his endowment $y_{jt}$, pays taxes $\tau_{jt}$ in real terms and consumes $c_{jt}$, subject of course to the cash constraint (2). Consequently, the budget constraint for the household is:

$$\omega_{jt+1} = (1+i_t)\omega_{jt} - i_t m_{jt} + P_t y_{jt} - P_t \tau_{jt} - P_t c_{jt}. \tag{4}$$

### *2.2. Aggregation, endowments and taxes*

Aggregate quantities are obtained by summing the various individual variables. There are $N_j - N_{j-1}$ agents in generation $j$. So for example $\Omega_t$ is given by:

$$\Omega_t = \sum_{j \leqslant t} (N_j - N_{j-1})\omega_{jt} \tag{5}$$

and similar formulas apply to $Y_t$, $C_t$, $M_t$, $B_t$, and $T_t$.

We also have to describe how aggregate endowments and taxes are distributed among households. We assume that all households have the same income and taxes, so that:

$$y_{jt} = y_t = \frac{Y_t}{N_t}, \qquad \tau_{jt} = \tau_t = \frac{T_t}{N_t}. \tag{6}$$

Taxes $T_t$ are determined by government policy (see Sections 3 and 9 below). As for output, we shall assume that output per head grows at the rate $\zeta$, so that:

$$\frac{y_t}{y_{t-1}} = \zeta, \qquad \frac{Y_t}{Y_{t-1}} = \zeta(1+n). \tag{7}$$

### *2.3. Government*

Now the other important part of the model is the government. The households' aggregate financial wealth $\Omega_t$ has as a counterpart an identical amount $\Omega_t$ of financial liabilities of the government. The evolution of these liabilities is described by the government's budget constraint:

$$\Omega_{t+1} = (1+i_t)B_t + M_t - P_t T_t = (1+i_t)\Omega_t - i_t M_t - P_t T_t. \tag{8}$$

### *2.4. Monetary policies*

As we indicated in the introduction, we shall study two types of monetary policies. The first is interest rate pegging, which consists in setting the nominal interest rate $i_t$ exogenously. We shall sometimes for the simplicity of exposition take the particular case where the interest rate is pegged at a constant value:

$$i_t = i_0. \tag{9}$$

The second type of policy we shall consider consists of "Taylor rules," through which the nominal interest rate responds to inflation:

$$i_t - i_0 = a(\pi_t - \pi_0), \quad a \geqslant 0, \tag{10}$$

where $\pi_0$ is the long-run rate of inflation and $i_0$ a target interest rate. The "Taylor principle" suggests that, for prices to be determinate, the coefficient $a$ should be greater than 1.

## 3. The dynamic equilibrium[8]

Households of generation $j$ maximize their utility (1) subject to their budget constraint (4). This yields the following Euler equation:

$$P_{t+1}c_{jt+1} = \beta(1+i_t)P_t c_{jt}. \tag{11}$$

As it turns out, when one aggregates consumption functions across generations, a new term appears due to the non-Ricardian character of the economy. It is shown indeed in the appendix that the following dynamic equation holds:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_t Y_t - (1-\beta)n\Omega_{t+1}. \tag{12}$$

Through the last term, which disappears if $n = 0$, accumulated financial wealth will now play an important role (we shall see an explanation in Section 7 below).

Since the model is non-Ricardian, the dynamics will depend on the actual tax policy, as it will notably influence the dynamics of $\Omega_t$. In order to simplify the dynamics below, we shall assume that the tax policy of the government consists in balancing the budget period by period.[9] Taxes will thus cover exactly interest payments on bonds:

$$P_t T_t = i_t B_t. \tag{13}$$

We may immediately note, using the government's budget constraint (8), that under this balanced budget policy total financial wealth will remain constant:

$$\Omega_t = \Omega \quad \text{for all } t. \tag{14}$$

The dynamic equation (12) then becomes:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_t Y_t - (1-\beta)n\Omega. \tag{15}$$

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[8] Since we shall concentrate on the problem of price determinacy, we shall not explore in this article the conditions for existence of an equilibrium. Such conditions are given in Weil (1987, 1991) for his original model. A self contained description of the conditions corresponding to the adaptation in this article is available from the author.

[9] A more general policy is considered in Section 9.

## 4. Traditional results

We shall now briefly review some traditional results on price determinacy under interest rate rules in the Ricardian setting. In the Ricardian model $n = 0$, and Eq. (15) becomes:

$$P_{t+1}Y_{t+1} = \beta(1 + i_t)P_t Y_t \tag{16}$$

which is the traditional aggregate dynamic equation.

### 4.1. Interest rate pegging

Let us start with interest rate pegging, and assume that the nominal interest rate $i_t$ is exogenously pegged. Equation (16) can be rewritten as:

$$\frac{P_{t+1}}{P_t} = \beta(1 + i_t)\frac{Y_t}{Y_{t+1}}. \tag{17}$$

This formula is homogeneous of degree one in prices, and so is the intertemporal budget constraint in the Ricardian case. These homogeneity properties clearly imply that we have nominal indeterminacy, which validates the result of Sargent and Wallace (1975).

### 4.2. The Taylor principle

Let us now consider more general interest rate rules of the form:

$$i_t - i_0 = a(\pi_t - \pi_0), \quad a \geqslant 0, \tag{18}$$

and loglinearize Eq. (16):

$$\pi_{t+1} = i_t + \mathrm{Log}(\beta/\zeta) - n. \tag{19}$$

Inserting (18) into (20), we obtain:

$$\pi_{t+1} = a(\pi_t - \pi_0) + i_0 + \mathrm{Log}(\beta/\zeta) - n \tag{20}$$

which can be rewritten as:

$$\pi_t = \frac{\pi_{t+1}}{a} + \frac{a\pi_0 - i_0 - \mathrm{Log}(\beta/\zeta) + n}{a}. \tag{21}$$

Clearly the inflation rate will be determinate if $a > 1$ (the Taylor principle). Since the past price is predetermined, a determinate inflation rate also means a determinate price.

Assuming that the determinacy condition $a > 1$ holds, we can solve (21) forward and we obtain:

$$\pi_t = \frac{a\pi_0 - i_0 - \mathrm{Log}(\beta/\zeta) + n}{a - 1}. \tag{22}$$

## 5. Determinacy under an interest rate peg

We now revert to the more general non-Ricardian framework, and consider the first problem we mentioned, that of a pure interest rate peg. As above, we shall study the Walrasian version of the model. Let us first assume that the interest rate is pegged at the value $i_0$. The dynamic system (15) is written:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_0)P_tY_t - (1-\beta)n\Omega. \tag{23}$$

In what follows it will be convenient to use nominal income $X_t$ as our working variable:

$$X_t = P_tY_t \tag{24}$$

so that (23) is rewritten as:

$$X_{t+1} = \beta(1+n)(1+i_0)X_t - (1-\beta)n\Omega. \tag{25}$$

Applying the conditions of Blanchard–Kahn (1980), we see that there is a unique solution in $X_t$ provided that:

$$\theta = \beta(1+n)(1+i_0) > 1 \tag{26}$$

and this solution is given by:

$$X_t = X_0 = \frac{(1-\beta)n\Omega}{\beta(1+n)(1+i_0)-1}. \tag{27}$$

Appendix A gives the solution in the case where the pegged interest rate is not constant in time.

## 6. The Taylor principle

Let us continue with the non-Ricardian model and turn to the Taylor principle. Loglinearizing Eq. (15), we obtain the following equation:

$$p_{t+1} + y_{t+1} = \theta(p_t + y_t) + \xi(i_t - i_0) \tag{28}$$

with:

$$\theta = \beta(1+n)(1+i_0) \quad \xi = \beta(1+n). \tag{29}$$

Combining with the equation giving the interest rate we obtain:

$$p_{t+1} = \theta p_t + \xi a\pi_t + \theta y_t - y_{t+1} - \xi a\pi_0. \tag{30}$$

This can actually be rewritten as a two dimensional dynamic system in inflation and the price level:

$$p_t = \pi_t + p_{t-1}, \tag{31}$$

$$\begin{aligned}\pi_{t+1} &= (\theta-1)p_t + \xi a\pi_t + \theta y_t - y_{t+1} - \xi a\pi_0 \\ &= (\theta-1+\xi a)\pi_t + (\theta-1)p_{t-1} + \theta y_t - y_{t+1} - \xi a\pi_0,\end{aligned} \tag{32}$$

or in matrix form, lagging variables one period:

$$\begin{bmatrix} \pi_t \\ p_{t-1} \end{bmatrix} = \begin{bmatrix} \theta - 1 + \xi a & \theta - 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ p_{t-2} \end{bmatrix} + \begin{bmatrix} \theta y_{t-1} - y_t - \xi a \pi_0 \\ 0 \end{bmatrix}. \tag{33}$$

The characteristic polynomial is:

$$\Phi(\mu) = \xi a(1-\mu) + \mu(\mu - \theta). \tag{34}$$

We have one predetermined variable (the past price) and one non predetermined (inflation). So there will be a determinate solution if the polynomial $\Phi(\mu)$ has one root of modulus smaller than 1, and the other greater than 1. So we compute:

$$\Phi(0) = \xi a \geqslant 0, \tag{35}$$

$$\Phi(1) = 1 - \theta. \tag{36}$$

Since furthermore $\Phi(\mu)$ goes to infinity when $\mu$ goes to infinity, we see that, if $\theta > 1$, we have one root between zero and one, and the other greater than one. So $\theta > 1$ is again a sufficient condition for determinacy, and whether $a$ is above or below 1 is not important anymore.

## 7. Economic interpretations

We just found that prices will be determined if $n > 0$ and condition (26), i.e. $\theta > 1$, is satisfied. This holds both for an interest rate peg and for a Taylor rule like (18). So it is time to give a few economic interpretations. There are actually several aspects.

### *7.1. The Pigou, or real balance effect*

When one looks at the dynamic equations (12) and (15), it appears clearly that a feature that drives most of the results is the presence of accumulated financial assets $\Omega_t$ in the dynamic equations. This is indeed a "nominal anchor," which is instrumental in tying down the value of prices. This presence of accumulated financial assets in various behavioral equations, and notably in the consumption function, has a history in the literature under the names of "Pigou effect" (Pigou, 1943, 1947) or "real balance effect" (Patinkin, 1965).

Why, in a world of rational expectations, does this Pigou effect occur when $n > 0$, and not for $n = 0$, has been very well explained by Weil (1991). Because of the intertemporal government budget constraint, when $n = 0$ the value of financial assets is exactly matched by discounted future tax liabilities, so that the effect on intertemporal wealth is zero. Now if instead $n > 0$, part of future taxes will be paid by yet unborn generations, so that part of $\Omega_t$ represents real wealth for the households alive.

### *7.2. Determinacy and the return on financial assets*

Now $n > 0$ creates a real balance effect. But this not the end of the story. Clearly this effect will be really operative only if the agents actually want to hold money and financial assets. And this is where condition (26) comes in. In order to interpret it, let us rewrite (26) under the following form:

$$\zeta(1+n)(1+i_0) > \frac{\zeta}{\beta}. \tag{37}$$

The left-hand side is the real rate of return on bonds. Indeed since $P_t Y_t = X_t$ is constant, and real resources grow at the rate $\zeta(1+n)$, in the steady state prices decrease at the rate $\zeta(1+n)$, and therefore the real rate of interest is $\zeta(1+n)(1+i_0)$.

Now $\zeta/\beta$ on the right-hand side of (37) is the real rate of return that would prevail in an economy with a single dynasty, a discount rate $\beta$, and where resources per head grow at the rate $\zeta$ (formula (7)). This is sometimes called the "autarkic" rate of return.

So conditions (26) or (37) essentially say that the real rate of return of bonds must be superior to the autarkic rate of return. We see that the above condition is very much similar to that found by Wallace (1980) for the viability of money in the traditional Samuelsonian (1958) overlapping generations model. There is an important difference, though: in Wallace (1980) the only financial store of value is money, so the rate of return condition concerns the return on money. Here this condition concerns the return on bonds, and accordingly the nominal interest rate plays an important role.

### 7.3. Suboptimality, price determinacy and the value of money

A theme that appears in the literature (and which is related to the previous one) is that money is valuable, and prices determinate, when the economy without money is Pareto suboptimal. This is notably studied in Wallace (1980) in the framework of the traditional Samuelsonian (1958) OLG model. Then, if the quantity of money stays constant, money will be valued and prices determinate if the "autarkic" equilibrium without money is Pareto suboptimal.

Here the relation between the suboptimality of the autarkic equilibrium and price determinacy becomes somewhat less clearcut. Indeed the condition for Pareto suboptimality of the autarkic equilibrium, is[10] $\beta(1+n) > 1$. Now the condition for price determinacy is $\beta(1+n)(1+i_0) > 1$, so the two conditions are not the same. In particular the above inequalities suggest that there are cases where the autarkic equilibrium is Pareto optimal and nevertheless prices are determinate. This will occur when:

$$\beta(1+n) < 1 \quad \text{and} \quad \beta(1+n)(1+i_0) > 1. \tag{38}$$

One can also find cases (Section 9 below) where the autarkic equilibrium is Pareto suboptimal, and prices nevertheless are not determinate.

So, although there is a clear conceptual connection between the two, there is not a one to one relationship between suboptimality and price determinacy.

[10] This is derived in Weil (1989) for the continuous time version of the model. A direct proof for the discrete time version we use here is available from the author.

### 7.4. A discontinuity

A particularly striking aspect of the results is the discontinuity in the determinacy conditions that arises when going from $n = 0$ to $n > 0$. An intuition is the following: just as OLG type models, the version of the Weil model we use in this article has 2 types of long-run equilibria:

(a) the ones where financial assets have positive real value (these are the ones studied in this article), and
(b) the ones where financial assets have zero real value ($\Omega_t / P_t = 0$).

There should be a continuity result between the Ricardian model and the equilibria of type (b) because they have a similar structure.[11] But such is not the case for the equilibria of type (a), so that the discontinuity needs not surprise us in this case.

## 8. The Taylor principle with a Phillips curve

So far we have studied the issue of price determinacy under the assumption of full market clearing. But the issue of price determinacy under interest rate rules has been very often studied in models with non clearing markets where output is demand determined and prices adjust partially according to a forward looking "Phillips curve" of the type:

$$\pi_t = \frac{1}{\alpha} E_t \pi_{t+1} + \delta y_t, \quad \alpha > 1,\ \delta > 0. \tag{39}$$

We want to show now that the results we obtained above in a Walrasian economy extend to this framework as well.

Phillips curves such as (39) are most often derived from a framework of contracts à la Calvo (1983).[12] Clearly the rigorous derivation of such a Phillips curve in our setting, would take us much too far.[13] So we shall simply take the Phillips curve (39) as given, and show that going from a Ricardian to a non-Ricardian framework leads again to major changes.

Again, the monetary authority uses an interest rate rule of the Taylor type:

$$i_t - i_0 = a(\pi_t - \pi_0). \tag{40}$$

In order to better highlight the differences, let us now begin with the Ricardian version of the model.

[11] In fact they have the same central dynamic equation. Compare Eq. (15) with $\Omega / P_t = 0$ and Eq. (16).

[12] They can be also derived from a model with convex costs of changing prices. See Rotemberg (1987) for an early derivation under both interpretations.

[13] Note indeed that, besides overlapping contracts, we have an infinity of households with different marginal utilities of income. Also the purpose of this article is to show that considering a non-Ricardian world leads to dramatic changes in the determinacy conditions. This point is made more clearly if we use the same nominal rigidity in both cases.

### 8.1. The Ricardian case

Output is now endogenous. Since it is demand determined, Eq. (16) is still valid. Loglinearizing it we obtain:

$$y_{t+1} = y_t + \operatorname{Log}\beta + (i_t - \pi_{t+1}). \tag{41}$$

Combining this with the interest rule (40) yields:

$$y_{t+1} = y_t + \operatorname{Log}\beta + i_0 + a(\pi_t - \pi_0) - \pi_{t+1}. \tag{42}$$

Equations (39) and (42) are rewritten, replacing $E_t\pi_{t+1}$ by $\pi_{t+1}$, since the model is deterministic:

$$\pi_{t+1} = \alpha(\pi_t - \delta y_t), \tag{43}$$

$$y_{t+1} = (1+\alpha\delta)y_t + (a-\alpha)\pi_t + \operatorname{Log}\beta + i_0 - a\pi_0. \tag{44}$$

This is written under matrix form:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1+\alpha\delta & a-\alpha \\ -\alpha\delta & \alpha \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \operatorname{Log}\beta + i_0 - a\pi_0 \\ 0 \end{bmatrix}. \tag{45}$$

The characteristic polynomial is:

$$\Phi(\mu) = (\mu-1)(\mu-\alpha) + \alpha\delta(a-\mu), \tag{46}$$

$$\Phi(0) = \alpha(1+\delta a) > 0, \tag{47}$$

$$\Phi(1) = \alpha\delta(a-1). \tag{48}$$

If $a < 1$, we have one root between 0 and 1. Since neither $y_t$ and $\pi_t$ are predetermined, this means that we have indeterminacy. On the other hand, if $a > 1$ the two roots have modulus greater than 1, and we have determinacy. We thus find again that the Taylor principle holds in this Ricardian framework.

### 8.2. The non-Ricardian case

Let us now move to the non-Ricardian economy. Equation (15) still holds. Loglinearizing it, we find that output, inflation and prices are linked by the following equation:

$$y_{t+1} + p_{t+1} = \theta(y_t + p_t) + \xi(i_t - i_0) \tag{49}$$

where the values of $\theta$ and $\xi$ are given in Eq. (29). We now express $y_{t+1}$, $\pi_{t+1}$ and $p_t$ as a function of the corresponding lagged variables:

$$p_t = \pi_t + p_{t-1}, \tag{50}$$

$$\pi_{t+1} = \alpha(\pi_t - \delta y_t), \tag{51}$$

$$y_{t+1} = (\alpha\delta + \theta)y_t + (\xi a - \alpha + \theta - 1)\pi_t + (\theta - 1)p_{t-1} - \xi a\pi_0, \tag{52}$$

or in matrix form (omitting the constants):

$$\begin{bmatrix} y_t \\ \pi_t \\ p_{t-1} \end{bmatrix} = \begin{bmatrix} \alpha\delta+\theta & \xi a - \alpha + \theta - 1 & \theta - 1 \\ -\alpha\delta & \alpha & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ p_{t-2} \end{bmatrix}. \tag{53}$$

The characteristic polynomial is:

$$\Phi(\mu) = (1-\mu)(\alpha-\mu)(\theta-\mu) + \alpha\delta\xi a(1-\mu) + \alpha\delta\mu(\mu-\theta). \tag{54}$$

We shall now show that $\theta > 1$ is again a sufficient condition for determinacy. There is one predetermined variable (the past price) and two non predetermined ones (output and inflation). So there will be a determinate solution if the polynomial $\Phi(\mu)$ has one root of modulus smaller than 1, and two roots of modulus greater than 1. Let us compute:

$$\Phi(0) = \alpha(\theta + \delta\xi a) > 0, \tag{55}$$

$$\Phi(1) = \alpha\delta(1-\theta). \tag{56}$$

So there is, assuming $\theta > 1$, one root between zero and one. Now the product of the three roots is $\Phi(0) = \alpha(\theta + \delta\xi a) > 1$. So the only possible case where the two remaining roots would not be of modulus greater than 1 would be that where we have two negative roots, one smaller than $-1$, one greater. In that case we would have $\Phi(-1) < 0$. This means that, together with $\theta > 1$, $\Phi(-1) > 0$ is a sufficient condition for determinacy. So we compute:

$$\Phi(-1) = 2(1+\alpha)(1+\theta) + 2\alpha\delta\xi a + \alpha\delta(1+\theta) > 0. \tag{57}$$

To summarize, if $\theta > 1$, we have one root between zero and one, and two roots of modulus greater than one, so that the inflation rate is determinate, and thus so is the price level.

## 9. Variable government liabilities

We shall now study a generalization of the fiscal policy (13) and assume that, instead of balancing the budget, the government engineers through taxes proportional expansions (or reductions) of its financial liabilities $\Omega_t$ (such an experiment was studied in Wallace, 1980), and we shall see how this affects the conditions for determinacy. More precisely we shall assume taxes of the form:

$$P_t T_t = i_t B_t + \rho\Omega_t, \quad \rho < 1. \tag{58}$$

As a result the evolution of $\Omega_t$ is given by:

$$\Omega_{t+1} = (1-\rho)\Omega_t. \tag{59}$$

Most of the analysis seen previously is still valid, and in particular Eq. (12) which we recall here:

$$P_{t+1}Y_{t+1} = \beta(1+n)(1+i_t)P_t Y_t - (1-\beta)n\Omega_{t+1}. \tag{60}$$

The dynamic system consists of Eqs. (59) and (60). In view of the homogeneity properties, this is actually a system in $P_t/\Omega_t$ and $Y_t$. Using (59), Eq. (60) can be rewritten as:

$$\frac{P_{t+1}Y_{t+1}}{\Omega_{t+1}} = \frac{\beta(1+n)(1+i_t)}{1-\rho}\frac{P_t Y_t}{\Omega_t} - (1-\beta)n. \tag{61}$$

We can first study the determinacy conditions for a pure interest rate peg $i_t = i_0$. Inserting this into (61), we see that the condition for determinacy is:

$$\beta(1+n)(1+i_0) > 1-\rho, \tag{62}$$

or $\theta > 1-\rho$. We may first note that this equation has an interpretation very similar to that of Eq. (26) that we saw before. Indeed it can be rewritten as:

$$\frac{\zeta(1+n)(1+i_0)}{1-\rho} > \frac{\zeta}{\beta}. \tag{63}$$

Since nominal assets are growing at the rate $1-\rho$, the long-run rate of inflation is $(1-\rho)(1+n)/\zeta$, so that the left-hand side is the real rate of return on financial assets, and the rest of the intuition given in Section 7.2 continues to hold.

We shall now see that the "expanded" condition (62) is actually sufficient for determinacy in all the non-Ricardian cases we have been considering in Sections 6 and 8. Let us indeed loglinearize Eq. (61). We obtain:

$$p_{t+1} + y_{t+1} - \omega_{t+1} = \frac{\theta}{1-\rho}(p_t + y_t - \omega_t) + \frac{\xi a}{1-\rho}(\pi_t - \pi_0) \tag{64}$$

where $\theta$ and $\xi$ are the same as in (29). We see that all the analysis we carried in the previous sections will be valid provided we replace $p_t$ by $p_t - \omega_t$ and the parameters $\theta$ and $\xi$ by $\theta/(1-\rho)$ and $\xi/(1-\rho)$ respectively.

Now we should note two things about condition (62). The first is that it shows most clearly the tradeoffs faced by the government on fiscal and monetary policy. Indeed a strict fiscal policy (high $\rho$) allows to lead a less rigorous monetary policy (low $i_0$), and the other way around.

The consideration of policies such as (58) also weakens the link between price determinacy and suboptimality. For example this introduces the possibility that the autarkic equilibrium is Pareto suboptimal, and prices nevertheless are not determinate (this will occur for negative enough $\rho$). This point was already made in Wallace (1980).

## 10. Conclusions

We have seen that going from a Ricardian to a non-Ricardian framework changes dramatically the conditions for price determinacy under interest rate rules. It is usually found in a Ricardian framework that interest rate pegging leads to nominal indeterminacy, and that a more than one to one response of interest rates to inflation leads to price determinacy.

We found instead that a strong response of the interest rate rule to inflation is not necessary for price determinacy, which can be achieved even under an interest rate peg. We identified sufficient conditions for determinacy (conditions (26) or (62)), which express that the real rate of return on nominal assets must be superior to the real rate of return that would prevail in the corresponding economy without financial assets. This condition ensures that agents will be actually willing to hold money and financial assets in the long run, obviously a critical condition if one wants money to have value, and prices to be determinate.

All the determinacy results in this article have been obtained using the Blanchard–Kahn (1980) criterion, which is the standard criterion in this domain. Less common criteria could possibly have been used. For example, Bénassy (2000) studied a model of interest pegging similar to that of Section 5 with a determinacy criterion based on transversality conditions, whereby prices are said to be determinate if only one trajectory satisfies the transversality conditions of the model. The determinacy conditions so obtained turn out to be very akin to conditions encountered in the fiscal theory of the price level. In particular to achieve determinacy the government should somehow pursue "explosive" fiscal policies. This is clearly not a policy one would recommend and this may cast some doubts on the adequacy of using that criterion in monetary environments like this one. The determinacy conditions we found in this article (like condition (26) for an interest rate peg) are clearly much more reasonable, which is why we used the Blanchard–Kahn criterion, like most writers in this field do.

## Acknowledgments

I wish to thank Gary Hansen, John Taylor and the Referees for their comments on earlier versions of this article. Of course, all remaining deficiencies are mine.

## Appendix A

### *A.1. Derivation of Eq. (12)*

We shall derive in this appendix the dynamic equation (12). Consider the household's budget equation (4). We assume that $i_t$ is strictly positive, so the household will always want to satisfy the "cash in advance" equation exactly. We thus have $m_{jt} = P_t c_{jt}$ and the budget constraint is rewritten as:

$$\omega_{jt+1} = (1 + i_t)\omega_{jt} + P_t y_t - P_t \tau_t - (1 + i_t) P_t c_{jt}. \tag{65}$$

In what follows we shall repeatedly aggregate discounted values. It is convenient to compute in monetary terms, and we shall thus use the following discount factors:

$$R_t = \prod_{s=0}^{t-1} \frac{1}{1 + i_s}, \quad R_0 = 1. \tag{66}$$

Applying the discount factors (66) to the budget constraint (65), it becomes:

$$R_{s+1}\omega_{js+1} = R_s \omega_{js} + R_{s+1} P_s (y_s - \tau_s) - R_s P_s c_{js}. \tag{67}$$

If we now aggregate all budget constraints (67) from time $t$ to infinity, and assume that $R_s \omega_{js}$ goes to zero as $s$ goes to infinity (this is the usual transversality condition), we obtain the intertemporal budget constraint of the household:

$$\sum_{s=t}^{\infty} R_s P_s c_{js} = R_t \omega_{jt} + \sum_{s=t}^{\infty} R_{s+1} P_s (y_s - \tau_s). \tag{68}$$

Now, maximizing utility function (1) subject to the intertemporal budget constraint (68) yields the following consumption function for a household $j$:

$$R_t P_t c_{jt} = (1-\beta)\left[R_t \omega_{jt} + \sum_{s=t}^{\infty} R_{s+1} P_s (y_s - \tau_s)\right]. \tag{69}$$

Summing this across the $N_t$ agents alive in period $t$, we obtain the aggregate consumption $C_t$:

$$R_t P_t C_t = (1-\beta)\left[R_t \Omega_t + N_t \sum_{s=t}^{\infty} R_{s+1} P_s (y_s - \tau_s)\right]. \tag{70}$$

In equilibrium we have $C_t = Y_t$, so the equilibrium equation is:

$$R_t P_t Y_t = (1-\beta)\left[R_t \Omega_t + N_t \sum_{s=t}^{\infty} R_{s+1} P_s (y_s - \tau_s)\right]. \tag{71}$$

Let us divide both sides by $N_t$ and use $Y_t = N_t y_t$:

$$R_t P_t y_t = (1-\beta)\left[\frac{R_t \Omega_t}{N_t} + \sum_{s=t}^{\infty} R_{s+1} P_s (y_s - \tau_s)\right]. \tag{72}$$

Let us rewrite this equation for $t+1$ and subtract it from (72). We obtain:

$$R_t P_t y_t - R_{t+1} P_{t+1} y_{t+1} = (1-\beta)\left[\frac{R_t \Omega_t}{N_t} - \frac{R_{t+1} \Omega_{t+1}}{N_{t+1}} + R_{t+1} P_t (y_t - \tau_t)\right]. \tag{73}$$

Now, let us multiply the government's budget equation (8) by $R_{t+1}/N_t$:

$$\frac{R_t \Omega_t}{N_t} = \frac{R_{t+1} \Omega_{t+1}}{N_t} + R_{t+1} P_t \tau_t + (R_t - R_{t+1}) P_t y_t. \tag{74}$$

Insert this into Eq. (73):

$$R_{t+1} P_{t+1} y_{t+1} = \beta R_t P_t y_t - (1-\beta)\left(\frac{1}{N_t} - \frac{1}{N_{t+1}}\right) R_{t+1} \Omega_{t+1} \tag{75}$$

and multiplying by $N_{t+1}/R_{t+1}$:

$$P_{t+1} Y_{t+1} = \beta \frac{N_{t+1}}{N_t} (1+i_t) P_t Y_t - (1-\beta)\left(\frac{N_{t+1}}{N_t} - 1\right) \Omega_{t+1}. \tag{76}$$

Assuming finally $N_{t+1}/N_t = 1+n$, we obtain:

$$P_{t+1} Y_{t+1} = \beta (1+n)(1+i_t) P_t Y_t - (1-\beta) n \Omega_{t+1} \tag{77}$$

which is Eq. (12).

### *A.2. Interest rate pegging with variable interest rates*

We shall consider here the case where the pegged interest rate can vary in time. Equation (25) is replaced by:

$$X_{t+1} = \beta (1+n)(1+i_t) X_t - (1-\beta) n \Omega. \tag{78}$$

This can be rewritten as:

$$X_t = \frac{X_{t+1} + (1-\beta)n\Omega}{\beta(1+n)(1+i_t)}. \tag{79}$$

A sufficient condition for determinacy is:

$$\beta(1+n)(1+i_t) > 1. \tag{80}$$

Using the discount factors (66), Eq. (79) can be rewritten as:

$$X_t = \frac{R_{t+1}}{R_t} \frac{X_{t+1} + (1-\beta)n\Omega}{\beta(1+n)}. \tag{81}$$

If condition (80) is satisfied, this can be integrated forward:

$$X_t = \frac{(1-\beta)n\Omega}{R_t} \sum_{i=1}^{\infty} \frac{R_{t+i}}{\beta^i (1+n)^i}. \tag{82}$$

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