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IS–LM and the multiplier: A dynamic general equilibrium model

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Abstract

We construct a dynamic general equilibrium model which displays the central features of the IS–LM model, and notably an income multiplier greater than one, so that crowding out does not occur. A key to this result is the conjunction of two features: price rigidities (as is usually expected), but also a non-Ricardian economy.
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1. Introduction

If one looks at the history of macroeconomics, one sees that two paradigms have dominated the profession. First, until the early seventies, the IS–LM model (Keynes, 1936; Hicks, 1937), then the dynamic stochastic general equilibrium (DSGE) paradigm (Lucas, 1972; Kydland and Prescott, 1982). For a while many people believed that these two paradigms were totally irreconcilable, notably due to the fact that early DSGE models, then called "real business cycles" (Long and Plosser, 1983), assumed full market clearing, whereas a central feature of IS–LM models was wage and price rigidities.

At some point, however, wage and price rigidities were introduced into DSGE models, which narrowed the gap between the two paradigms. Some authors constructed DSGE models displaying some features of the IS–LM model (see for example McCallum and Nelson, 1999, and references therein).

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The integration is not yet complete, however, as some important features of the IS–LM model have not made fully their way into DSGE models. Notably one of the most emblematic features of IS–LM, an income multiplier greater than 1, has been difficult to obtain in DSGE models as there is usually crowding out of consumption by government spending (see for example Fatas and Mihov, 2001; Collard and Dellas, 2005).

So the purpose of this paper is to construct a DSGE model which (1) will display the central features of the IS–LM model and (2) will notably allow to obtain a multiplier larger than one.

2. The model

It was shown in Bénassy (2007) that in order to obtain multiplier effects it was important not only to have price or wage rigidities, but also to consider non Ricardian economies.[1] By non Ricardian economies we mean, as in Barro (1974), economies where, due for example to the birth of new agents over time, Ricardian equivalence does not hold.

So we shall consider a particularly simple non Ricardian economy, a monetary overlapping generations economy à la Samuelson (1958). In each period t three markets are open: the goods market at price P_t, the labor market at wage W_t and the bonds market at the interest rate i_t.

The private sector consists of firms and households. The representative firm in period t has a production function:

$$Y_t = Z_t L_t \tag{1}$$

where L_t is employment, Z_t a productivity shock. Households live for two periods. Households born in t work L_t, consume C_{1t} and hold a quantity of money M_{1t} in period t. In period $t+1$ they consume C_{2t+1} and hold money M_{2t+1}. They maximize the expected value of their utility U_t:

$$U_t = \alpha \mathrm{Log} V_{1t} + (1-\alpha)\mathrm{Log} V_{2t+1} - L_t/a \tag{2}$$

$$V_{it} = V(C_{it}, M_{it}/P_t) \quad i = 1, 2 \tag{3}$$

where the function V is homogeneous of degree one in its two arguments and M_{it} is money held at the end of period t (this is the traditional "money in the utility function" specification).

Let us now turn to government, which uses monetary and fiscal policy. Monetary policy consists in choosing the interest rate i_t. Note that, although the interest rate is the policy tool, it may be manipulated so as to target a monetary aggregate, such as the quantity of money.

The second tool, fiscal policy, consists in choosing the levels of government spending G_t and taxes T_t in real terms. We assume that taxes are levied on the young households, proportionately to their total income[2] at the rate τ_t. The tax rate τ_t is chosen so as to yield a real amount of taxes T_t:

$$\tau_t(W_t L_t + \Pi_t) = \tau_t P_t Y_t = P_t T_t \tag{4}$$

[1] In Bénassy (2007) prices are totally rigid and money demand is insensitive to the interest rate. These restrictions are lifted here so as to obtain a model as close as possible to the IS–LM model.

[2] Proportional taxes are used because lump sum taxes create income effects on labor supply which make some equations look less "natural". But the central results are the same.

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Let us finally examine the budget constraints. The old household starts period t with total financial assets Ω_t. He spends P_tC_{2t}, and splits the rest between bonds B_{2t} and money M_{2t}, so that $M_{2t}+B_{2t}=\Omega_t-P_tC_{2t}$. If he lived through another period his wealth next period would be:

$$(1+i_t)B_{2t}+M_{2t}=(1+i_t)(\Omega_t-P_tC_{2t})-i_tM_{2t} \tag{5}$$

But since he is not going to be alive next period, he wants this wealth to be zero, so that his budget constraint is:

$$P_tC_{2t}+i_tM_{2t}/(1+i_t)=\Omega_t \tag{6}$$

Consider now the young household born in t. He starts life with no assets, but receives wage income W_tL_t and profit income $\Pi_t=P_tY_t-W_tL_t$, both taxed at the rate τ_t. He also spends P_tC_{1t}, so that the sum of money M_{1t} and bonds B_{1t} is:

$$M_{1t}+B_{1t}=(1-\tau_t)(W_tL_t+\Pi_t)-P_tC_{1t} \tag{7}$$

And his financial holdings at the beginning of next period Ω_{t+1} will be $\Omega_{t+1}=(1+i_t)B_{1t}+M_{1t}$, i.e.:

$$\Omega_{t+1}=(1+i_t)[(1-\tau_t)(W_tL_t+\Pi_t)-P_tC_{1t}]-i_tM_{1t} \tag{8}$$

3. The demand for money and the LM curve

Budget constraints and maximization programs are particularly easy to study if we use the intermediate variable "total spending" S_{it} defined as:

$$S_{it}=P_tC_{it}+j_tM_{it} \tag{9}$$

with:

$$j_t=i_t/(1+i_t) \tag{10}$$

The second term on the r.h.s. of Eq. (9) represents the value of interest foregone from holding money instead of bonds. From the preceding budget constraints we see that optimal choices of the households can be broken in two: (a) the choice of S_{it} ($i=1, 2$) and (b) the allocation of S_{it} between consumption and money. Let us start with this second problem: For a given value of S_{it} the household, whether young or old, maximizes the value of V_{it}, i.e. he solves the following problem:

$$\text{Maximize } V_{it}=V(C_{it},M_{it}/P_t) \quad \text{s.t.} \quad P_tC_{it}+j_tM_{it}=S_{it}$$

The first order condition is:

$$V_2/V_1=j_t \tag{11}$$

where V_1 and V_2 are the first and second partial derivatives of V. In view of the homogeneity of degree one of the function V, Eq. (11) defines a "demand for money" for the agent, which has the following form:

$$M_{it}/P_tC_{it}=\Phi(j_t) \quad \Phi'\leq 0 \tag{12}$$

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Now combining Eqs. (9) and (12) we obtain the following relation between "total spending" and consumption:

$$S_{it} = P_t C_{it} + j_t M_{it} = \zeta_t P_t C_{it} \tag{13}$$

$$\zeta_t = 1 + j_t \Phi(j_t) > 1 \tag{14}$$

Let us now move to aggregate values: $C_t = C_{1t} + C_{2t}$ and $M_t = M_{1t} + M_{2t}$. Aggregating relation (12) for young and old ($i = 1$ and 2), we obtain the total demand for money:

$$\frac{M_t}{P_t C_t} = \Phi(j_t) \quad \Phi' \leq 0 \tag{15}$$

in which, since $j_t = i_t/(1+i_t)$ is strictly increasing in i_t, we recognize a demand for money very close to the traditional LM curve (such a LM curve appears already, for example, in McCallum and Nelson, 1999).

4. Walrasian equilibrium

Let us start with old households, who enter period t with a financial wealth Ω_t. Combining Eqs. (6), (10) and (13) we find the consumption of the old:

$$C_{2t} = \Omega_t / \zeta_t P_t \tag{16}$$

The young household is subject to the intertemporal budget constraint (8), which, in view of (13), becomes:

$$\Omega_{t+1} = (1 + i_t)[(1-\tau_t)(W_t L_t + \Pi_t) - \zeta_t P_t C_{1t}] \tag{17}$$

In the first period of his life the household maximizes the expectation of his utility (2) subject to the budget constraint (17). We note, from Eq. (16), that C_{2t+1} is proportional to Ω_{t+1}. So the young household solves:

$$\text{Maximize } \alpha \text{Log} C_{1t} + (1-\alpha) \text{Log} \Omega_{t+1} - L_t/a \quad \text{s.t.} \quad \Omega_{t+1} = (1 + i_t)[(1-\tau_t)(W_t L_t + \Pi_t) - \zeta_t P_t C_{1t}]$$

The first order conditions for this program yield:

$$\zeta_t P_t C_{1t} = \alpha(1-\tau_t)(W_t L_t + \Pi_t) = \alpha(1-\tau_t) P_t Y_t = \alpha(P_t Y_t - P_t T_t) \tag{18}$$

$$a W_t = P_t Y_t \tag{19}$$

Turning to firms, the real wage must be equal to marginal productivity:

$$W_t = P_t Z_t \tag{20}$$

Finally the equation of equilibrium on the goods market is:

$$Y_t = C_t + G_t = C_{1t} + C_{2t} + G_t \tag{21}$$

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Combining Eqs. (16), (18)–(21) we find the values of Walrasian output, employment, consumption and price:

$$Y_t = aZ_t \quad L_t = a \quad C_t = aZ_t - G_t \tag{22}$$

$$P_t^* = \frac{\Omega_t}{(\zeta_t - \alpha)aZ_t - \zeta_t G_t + \alpha T_t} \tag{23}$$

We note that an increase in government spending G_t decreases C_t by an equal amount (Eq. (22)), the traditional "crowding out" effect.

5. The IS–LM system

Traditionally the IS–LM system is associated with price rigidities, wage rigidities, or both. So we shall look for equations that are valid in all these cases.

If wages are sticky Eq. (19) is not valid anymore. If prices are sticky Eq. (20) is not valid. But Eqs. (16), (18) and (21) are valid in all circumstances. Combining them we obtain:

$$Y_t = \frac{1}{\zeta_t - \alpha}\left(\frac{\Omega_t}{P_t} + \zeta_t G_t - \alpha T_t\right) \tag{24}$$

which looks very much like an *IS* curve. This has to be supplemented by the *LM* curve (Eq. (15)). Eqs. (15) and (24) form a system that is highly similar to the *IS–LM* system. We shall now investigate whether, as *IS–LM*, this system can display a strong multiplier effect.

6. Preset prices and the multiplier

We shall now introduce a traditional price rigidity, preset prices in the tradition of Gray (1976):

$$\frac{1}{P_t} = E_{t-1}\frac{1}{P_t^*} \tag{25}$$

We still have to specify how monetary policy is operated. There are two traditional polar ways, setting the interest rate or setting the money supply, and we study them in turn.

6.1. Setting the interest rate

We first assume that the government sets a given interest rate ($i_t = i$), letting the quantity of money adjust endogenously. As a consequence ζ_t is constant and equal to ζ. Combining Eqs. (23) and (25) and $\zeta_t = \zeta$ we find:

$$\frac{1}{P_t} = E_{t-1}\frac{(\zeta - \alpha)aZ_t - \zeta G_t + \alpha T_t}{\Omega_t} \tag{26}$$

Inserting this into Eq. (24) with $\zeta_t = \zeta$ we obtain:

$$Y_t = aE_{t-1}Z_t + \frac{\zeta}{\zeta - \alpha}(G_t - E_{t-1}G_t) - \frac{\alpha}{\zeta - \alpha}(T_t - E_{t-1}T_t) \tag{27}$$

We see that the multiplier on G_t, $\zeta/(\zeta - \alpha)$, is always greater than 1.

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6.2. Setting the money supply

We now study how the analysis is modified when the government controls the money supply instead of the interest rate. Since we are actually interested in the sign of the consumption response, let us rewrite the *IS* curve (24), deducting G_t from both sides, as:

$$C_t = \frac{1}{\zeta_t - \alpha}\left(\frac{\Omega_t}{P_t} + \alpha G_t - \alpha T_t\right) \tag{28}$$

Let us differentiate Eqs. (14), (15) and (28), calling ϕ the absolute value of the elasticity of Φ:

$$\mathrm{d}\zeta_t = \Phi(j_t)(1-\phi)\mathrm{d}j_t \tag{29}$$

$$\mathrm{d}C_t/C_t = \phi \mathrm{d}j_t/j_t \tag{30}$$

$$(\zeta_t - \alpha)\mathrm{d}C_t = \alpha \mathrm{d}G_t - C_t \mathrm{d}\zeta_t \tag{31}$$

Combining Eqs. (29)–(31) we obtain:

$$\frac{\mathrm{d}C_t}{\mathrm{d}G_t} = \frac{\alpha\phi}{\phi(1-\alpha) + (\zeta_t - 1)} \tag{32}$$

We see that consumption is again an increasing function of G_t. The multiplier is increasing in ϕ.

7. Partial price flexibility

Let us now assume that the current price is given by:

$$\frac{1}{P_t} = \gamma E_{t-1}\frac{1}{P_t^*} + (1-\gamma)\frac{1}{P_t^*} \tag{33}$$

For $\gamma = 0$ the price is fully flexible, for $\gamma = 1$ we obtain the "Gray contract" above. We shall study here the interest rate setting case. Combining Eqs. (23) and (33) we find:

$$\frac{1}{P_t} = (1-\gamma)\frac{(\zeta-\alpha)aZ_t - \zeta G_t + \alpha T_t}{\Omega_t} + \gamma E_{t-1}\frac{(\zeta-\alpha)aZ_t - \zeta G_t + \alpha T_t}{\Omega_t} \tag{34}$$

Inserting this into Eq. (24) with $\zeta_t = \zeta$ we obtain:

$$Y_t = (1-\gamma)aZ_t + \gamma a E_{t-1}Z_t + \frac{\gamma\zeta}{\zeta-\alpha}(G_t - E_{t-1}G_t) - \frac{\gamma\alpha}{\zeta-\alpha}(T_t - E_{t-1}T_t) \tag{35}$$

We see that the multiplier on G_t, $\gamma\zeta/(\zeta-\alpha)$, is greater than 1 if:

$$\alpha > \zeta(1-\gamma) \tag{36}$$

As should be expected from traditional Keynesian analysis, price flexibility (i.e. a lower γ) decreases the multiplier.

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8. Conclusion

We saw that by adequately combining price rigidities and a non Ricardian framework we could obtain, in a model looking very much like the traditional IS–LM, an income multiplier greater than one, or in other words that an increase in government spending could lead to an increase in private consumption. This was done in a dynamic stochastic general equilibrium model with maximizing agents.

Although it can be greater than one, the multiplier is not always so, and using a rigorous model allowed to specify the range of parameters for which no crowding out occurs. We studied notably the degree of price rigidity, and whether monetary policy is implemented by controlling interest rates or the supply of money.

It is thus comforting to know that the two paradigms that dominated macroeconomics for so long are, after all, not contradictory.

References

Barro, Robert J., 1974. Are government bonds net wealth? Journal of Political Economy 82, 1095–1117.
Bénassy, Jean-Pascal, 2007. Ricardian equivalence and the intertemporal Keynesian multiplier. Economics Letters 94, 118–123.
Collard, Fabrice, Dellas, Harris, 2005. Poole in the new Keynesian model. European Economic Review 49, 887–907.
Fatas, Antonio, Mihov, Ilian, 2001. The Effects of Fiscal Policy on Consumption and Employment: Theory and Evidence. INSEAD.
Gray, Jo-Anna, 1976. Wage indexation: a macroeconomic approach. Journal of Monetary Economics 2, 221–235.
Hicks, John R., 1937. Mr. Keynes and the "classics": a suggested interpretation. Econometrica 5, 147–159.
Keynes, John Maynard, 1936. The General Theory of Employment, Interest and Money. Harcourt Brace, New York.
Kydland, Finn E., Prescott, Edward, 1982. Time to build and aggregate fluctuations. Econometrica 50, 1345–1370.
Long, John B., Plosser, Charles I., 1983. Real business cycles. Journal of Political Economy 91, 39–69.
Lucas Jr., Robert E., 1972. Expectations and the neutrality of money. Journal of Economic Theory 4, 103–124.
McCallum, Bennett T., Nelson, Edward, 1999. An optimizing IS–LM specification for monetary policy and business cycle analysis. Journal of Money, Credit and Banking 31, 296–316.
Samuelson, Paul A., 1958. An exact consumption–loan model of interest with or without the social contrivance of money. Journal of Political Economy 66, 467–482.

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