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Employment Targeting

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Abstract

Many recent discussions on the conduct of monetary policy through interest rate rules have given a very central role to inflation, both as an objective and as an intermediate instrument. We want to show that other variables like employment can be as important or even more. For that we construct a dynamic stochastic general equilibrium (DSGE) model where the economy is subject to demand and supply shocks. We compute closed form solutions for the optimal interest rate rules and find that they can be function of employment only, which then dominates inflation for use in the policy rule.

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1 Introduction

In recent years both practical developments and academic discussions on the conduct of monetary policy have given a very central role to inflation, both as an objective and as an instrument. For example, whereas the Fed takes interest rate decisions on the basis of several objectives, including inflation, employment and growth, the more recently established European Central Bank is supposed to focus only on the fight against inflation. On the academic side, following Taylor's (1993) influential article, many authors have studied interest rate rules where central bank policy is notably function of inflation¹. A particularly scrutinized issue has been the famous "Taylor principle", according to which the nominal interest rate should respond more than one for one to inflation². This principle is supposed to be the key to both optimality and price determinacy.

Is this concentration on inflation warranted? We want to scrutinize this issue in a dynamic stochastic general equilibrium (DSGE) model where the economy is subject to both demand and supply shocks. We shall develop a simple model for which we will be able to compute closed form solutions for the optimal interest rate rules.

We shall find out that the optimal interest rate policy can be implemented as a function of employment as an instrument. Using inflation as an instrument would lead to much lower utility.

2 The model

2.1 The agents

We shall consider a monetary overlapping generations model (Samuelson, 1958) with production. The economy includes representative firms and households, and the government.

Households of generation t live for two periods, work N_t and consume C_{1t} in period t , consume C_{2t+1} in period $t + 1$. They maximize the expected value of the following two period utility:

$$U_t = \alpha_t \text{Log}C_{1t} + \text{Log}C_{2t+1} - (1 + \alpha_t) N_t \quad (1)$$

¹The original Taylor contribution introduced both inflation and output as arguments of the interest rate rule, but in subsequent writings the role of output has been overshadowed by that of inflation.

²The coefficient can actually be lower if the interest rate responds to other variables, such as the output gap. See, for example, Woodford (2003).

where $\alpha_t > 0$ is a demand shock. Households are submitted in each period of their life to a “cash in advance” constraint:

$$M_{1t} \geq P_t C_{1t} \quad M_{2t+1} \geq P_{t+1} C_{2t+1} \quad (2)$$

The total quantity of money is $M_t = M_{1t} + M_{2t}$. Since the young household starts his life without any asset, he has to borrow $P_t C_{1t}$ from the bank at the interest rate i_t in order to satisfy the cash in advance constraint. Consequently the bank makes profits Λ_t , equal to:

$$\Lambda_t = i_t P_t C_{1t} \quad (3)$$

To simplify calculations we assume that these profits Λ_t are redistributed lump-sum to the young households.

The representative firm in period t produces output Y_t with labor N_t via the production function:

$$Y_t = Z_t N_t \quad (4)$$

where Z_t is a technological shock common to all firms. We assume that the firms belong to the young households, to which they distribute their profits, if any.

2.2 Monetary policy and the optimality criterion

The central bank has essentially one policy instrument, the nominal interest rate i_t . Monetary policy will be represented as an “interest rate rule” expressing formally how the nominal interest rate responds to various observed variables.

In order to evaluate the optimality properties of potential interest rate policies, we shall use the criterion proposed by Samuelson for the overlapping generations model (Samuelson, 1967, 1968, Abel 1987) and assume that in period t the government maximizes the function V_t :

$$V_t = E_t \sum_{s=t-1}^{\infty} \beta^{s-t} U_s \quad (5)$$

The sum starts at $s = t - 1$ because the old household born in $t - 1$ is still alive in t . Rearranging the terms in the infinite sum (5), we find that, up to a constant, the criterion V_t can be rewritten under the more convenient form:

$$V_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \Delta_s \quad (6)$$

with:

$$\Delta_t = \alpha_t \text{Log} C_{1t} + \frac{1}{\beta} \text{Log} C_{2t} - (1 + \alpha_t) N_t \quad (7)$$

3 Market equilibrium

For the policy evaluations that will follow, we need to characterize first the market equilibrium.

Consider first the problem of the old households in period t . We denote by Ω_t the financial wealth that the old households own at the beginning of period t . With a hundred percent cash in advance constraint (formula 2), their consumption C_{2t} is given by:

$$P_t C_{2t} = \Omega_t \quad (8)$$

Now let us write the maximization program of the young household born in t . When young, the representative household receives wages $W_t N_t$, firms' profits $\Psi_t = P_t Y_t - W_t N_t$ and central bank profits Λ_t . If he consumes C_{1t} in the first period of his life, he will have accumulated at the end of period t a financial wealth Ω_{t+1} :

$$\Omega_{t+1} = (W_t N_t + \Psi_t + \Lambda_t) - (1 + i_t) P_t C_{1t} \quad (9)$$

In view of (8), the expected value of $\text{Log} C_{2t+1}$ is, up to a constant, equal to $\text{Log} \Omega_{t+1}$, so that the expected utility maximization program of the young household in the first period boils down to choosing C_{1t} and N_t so as to solve:

$$\text{Maximize } \alpha_t \text{Log} C_{1t} + \text{Log} \Omega_{t+1} - (1 + \alpha_t) N_t \quad \text{s.t.}$$

$$\Omega_{t+1} = (W_t N_t + \Psi_t + \Lambda_t) - (1 + i_t) P_t C_{1t}$$

The first order conditions with respect to C_{1t} and N_t are:

$$P_t C_{1t} = \frac{\alpha_t}{1 + \alpha_t} \frac{W_t N_t + \Psi_t + \Lambda_t}{1 + i_t} \quad (10)$$

$$W_t = W_t N_t + \Psi_t + \Lambda_t \quad (11)$$

Now the equilibrium condition on the goods market says that total consumption must equal production:

$$C_{1t} + C_{2t} = Y_t = Z_t N_t \quad (12)$$

Using the definition of profits $\Psi_t = P_t Y_t - W_t N_t$, and combining (3), (8), (10), (11) and (12), we obtain, instead of (10) and (11), the simpler expressions:

$$P_t C_{1t} = \frac{\alpha_t \Omega_t}{1 + i_t} \quad (13)$$

$$W_t = (1 + \alpha_t) \Omega_t \quad (14)$$

Finally if firms are on their supply curve the price is equal to marginal cost:

$$P_t = \frac{W_t}{Z_t} \quad (15)$$

We should finally note that, since central bank profits Λ_t are redistributed to the households, Ω_t will remain constant in time. Indeed, combining (3), (8), (9) and (12), we obtain:

$$\Omega_{t+1} = P_t Y_t - (1 + i_t) P_t C_{1t} + \Lambda_t = P_t C_{2t} = \Omega_t = \Omega \quad (16)$$

For what follows we need the expressions of the Walrasian wage and price P_t^* and W_t^* . From (14) and (15) these are:

$$W_t^* = (1 + \alpha_t) \Omega_t \quad (17)$$

$$P_t^* = \frac{(1 + \alpha_t) \Omega_t}{Z_t} \quad (18)$$

4 Optimal interest policy: the Walrasian case

As a benchmark, we shall now compute the optimal interest rate policy in the case where all markets clear.

Proposition 1: *Under Walrasian prices and wages the optimal interest rate rule is:*

$$i_t = 0 \quad (19)$$

Proof: The central bank must choose the interest rate so as to maximize, in each period and for each value of the shocks:

$$\Delta_t = \alpha_t \text{Log} C_{1t} + \frac{1}{\beta} \text{Log} C_{2t} - (1 + \alpha_t) N_t \quad (20)$$

or, using (12):

$$\Delta_t = \alpha_t \text{Log} C_{1t} + \frac{1}{\beta} \text{Log} C_{2t} - (1 + \alpha_t) \frac{C_{1t} + C_{2t}}{Z_t} \quad (21)$$

We first note that C_{2t} (equation 8) does not depend on the interest rate, since P_t^* does not. Combining (13) and (18) we find that C_{1t} is given by:

$$C_{1t} = \frac{\alpha_t}{1 + \alpha_t} \frac{Z_t}{1 + i_t} \quad (22)$$

Inserting this value of C_{1t} into (21) and maximizing Δ_t with respect to i_t yields immediately (19). Q.E.D.

We see that, in the Walrasian case, no matter what are the shocks, the interest rate should remain totally unresponsive and equal to zero. This is actually the famous ‘‘Friedman rule’’ (Friedman, 1969).

5 Preset wages and the optimal rule

We will now move to the study of economies with nominal rigidities. We shall begin assuming preset wages, and make the assumption, traditional in the literature since Gray (1976), that the preset wage is equal to the expected value of the Walrasian wage³, i.e.:

$$W_t = E_{t-1} W_t^* \quad (23)$$

In view of (17) this yields:

$$W_t = E_{t-1} [(1 + \alpha_t) \Omega_t] = (1 + \alpha_a) \Omega_t \quad (24)$$

where:

$$\alpha_a = E(\alpha_t) \quad (25)$$

³A number of authors have given microfoundations to similar formulas in a framework of monopolistic competition. See, for example, Bénassy (2002).

5.1 Equilibrium conditions

Since the wage is preset, equation (14), which reflects the labor supply behavior of the household, does not hold anymore, but the other equilibrium conditions (8), (12), (13) and (15) are still valid. Combining them with the expression of the preset wage (24) we find:

$$C_{1t} = \frac{\alpha_t Z_t}{(1 + \alpha_a)(1 + i_t)} \quad (26)$$

$$C_{2t} = \frac{Z_t}{1 + \alpha_a} \quad (27)$$

$$N_t = \frac{\alpha_t}{(1 + \alpha_a)(1 + i_t)} + \frac{1}{1 + \alpha_a} \quad (28)$$

5.2 The optimal interest rate rule

We shall now derive the optimal interest rate rule as a function of the shocks:

Proposition 2: *Under preset wages the optimal interest rate rule is:*

$$1 + i_t = \max \left(1, \frac{1 + \alpha_t}{1 + \alpha_a} \right) \quad (29)$$

Proof: We have to maximize for all values of the shocks the quantity:

$$\Delta_t = \alpha_t \text{Log} C_{1t} + \frac{1}{\beta} \text{Log} C_{2t} - (1 + \alpha_t) N_t \quad (30)$$

which, using (26), (27) and (28), is equal to:

$$\begin{aligned} \Delta_t = & \alpha_t \text{Log} \left[\frac{\alpha_t Z_t}{(1 + \alpha_a)(1 + i_t)} \right] + \frac{1}{\beta} \text{Log} \left(\frac{Z_t}{1 + \alpha_a} \right) \\ & - (1 + \alpha_t) \left[\frac{\alpha_t}{(1 + \alpha_a)(1 + i_t)} + \frac{1}{1 + \alpha_a} \right] \end{aligned} \quad (31)$$

Maximizing (31) with respect to i_t , and taking into account the fact that i_t must be positive, we find (29). Q.E.D.

We should note that rule (29) does not allow the economy to reach a “first best” situation. But it is the optimal rule subject to the constraint that wages are preset.

6 Employment as an instrument

Now generally the shocks are not observable and the policy maker must use variables that will act as surrogates for the shocks (see, for example, Bénassy, 2003). Traditionally in this literature inflation is the principal instrument. We shall now show that the level of employment can be used as a perfect surrogate, whereas the use of inflation would lead to a suboptimal rule.

Proposition 3: *Under preset wages, if employment N_t is used as an instrument, the optimal interest rate rule is:*

$$\frac{i_t}{1+i_t} = \max[0, (1+\alpha_a)(N_t-1)] \quad (32)$$

Rule (32) allows to reach the same level of utility as rule (29).

Proof: If the optimal policy (29) is used, we can compute, combining (28) and (29), the level of employment:

$$N_t = \min\left(\frac{\alpha_t}{1+\alpha_t} + \frac{1}{1+\alpha_a}, \frac{\alpha_t}{1+\alpha_a} + \frac{1}{1+\alpha_a}\right) \quad (33)$$

Inverting this relation, we can deduce the value of the demand shock α_t from N_t :

$$\frac{1}{1+\alpha_t} = \min\left[1 - N_t + \frac{1}{1+\alpha_a}, \frac{1}{(1+\alpha_a)N_t}\right] \quad (34)$$

The optimal rule (29) can be rewritten:

$$\frac{1}{1+i_t} = \min\left(1, \frac{1+\alpha_a}{1+\alpha_t}\right) \quad (35)$$

We insert the value of $1/(1+\alpha_t)$ in (34) into the optimal rule (29), which yields:

$$\frac{1}{1+i_t} = \min\left[1, (1+\alpha_a)(1-N_t) + 1, \frac{1}{N_t}\right] \quad (36)$$

It is easy to check, using the fact that $N_t > 1/(1+\alpha_a)$ (equation 33), that the third term is always smaller than the minimum of the two first ones, so that we can suppress it. Subtracting from 1 we obtain the optimal rule (32). Furthermore, since there is a one to one relation between N_t and α_t (equation 33), this rule allows to reach the same utility as the optimal rule in proposition 2. Q.E.D.

7 Preset prices

We will now turn to the case of preset prices, and assume that the preset price is equal to the expected value of the Walrasian price, i.e.:

$$P_t = E_{t-1}P_t^* \quad (37)$$

which, in view of the value of the Walrasian price (18), yields:

$$P_t = E_{t-1} \left[\frac{(1 + \alpha_t) \Omega_t}{Z_t} \right] = \frac{(1 + \alpha_a) \Omega_t}{Z_a} \quad (38)$$

where:

$$\frac{1}{Z_a} = E \left(\frac{1}{Z_t} \right) \quad (39)$$

7.1 Equilibrium conditions

Since the price is preset, equation (15) does not hold anymore, but the other equilibrium conditions (8), (12), (13) and (14) are still valid. Combining them with the expression of the preset price (38) we find:

$$C_{1t} = \frac{\alpha_t Z_a}{(1 + i_t)(1 + \alpha_a)} \quad (40)$$

$$C_{2t} = \frac{Z_a}{1 + \alpha_a} \quad (41)$$

$$N_t = \frac{Z_a}{(1 + \alpha_a) Z_t} \left(\frac{\alpha_t}{1 + i_t} + 1 \right) \quad (42)$$

7.2 The optimal interest rule

We shall now derive the optimal interest rate rule as a function of the shocks:

Proposition 4: *Under preset prices the optimal interest rate rule is:*

$$1 + i_t = \max \left[1, \frac{(1 + \alpha_t) Z_a}{(1 + \alpha_a) Z_t} \right] \quad (43)$$

Proof: We have to maximize for all values of the shocks the quantity:

$$\Delta_t = \alpha_t \text{Log} C_{1t} + \frac{1}{\beta} \text{Log} C_{2t} - (1 + \alpha_t) N_t \quad (44)$$

which, using (40), (41) and (42), is equal to:

$$\begin{aligned} \Delta_t = & \alpha_t \text{Log} \left[\frac{\alpha_t Z_a}{(1 + \alpha_a)(1 + i_t)} \right] + \frac{1}{\beta} \text{Log} \left(\frac{Z_a}{1 + \alpha_a} \right) \\ & - (1 + \alpha_t) \left[\frac{Z_a}{(1 + \alpha_a) Z_t} \left(\frac{\alpha_t}{1 + i_t} + 1 \right) \right] \end{aligned} \quad (45)$$

Maximizing (45) with respect to i_t , and taking into account the fact that i_t must be positive, we find (43). Q.E.D.

7.3 Employment as an instrument

We shall now see that we can express the optimal interest rate rule (43) using employment as an instrument:

Proposition 5: *Under preset prices, if employment N_t is used as an instrument, the optimal interest rate rule is:*

$$\frac{i_t}{1 + i_t} = \max \left[0, \frac{(1 + \alpha_a) Z_t}{Z_a} (N_t - 1) \right] \quad (46)$$

Rule (46) allows to reach the same level of utility as rule (43).

Proof: If the optimal rule (43) is used, we obtain the level of employment N_t , inserting (43) into (42):

$$N_t = \min \left[\frac{\alpha_t}{1 + \alpha_t} + \frac{Z_a}{(1 + \alpha_a) Z_t}, \frac{\alpha_t Z_a}{(1 + \alpha_a) Z_t} + \frac{Z_a}{(1 + \alpha_a) Z_t} \right] \quad (47)$$

We can derive the value of the demand shock from N_t and Z_t :

$$\frac{1}{1 + \alpha_t} = \min \left[1 - N_t + \frac{Z_a}{(1 + \alpha_a) Z_t}, \frac{Z_a}{(1 + \alpha_a) Z_t N_t} \right] \quad (48)$$

The optimal rule (43) can be rewritten:

$$\frac{1}{1 + i_t} = \min \left[1, \frac{(1 + \alpha_a) Z_t}{(1 + \alpha_t) Z_a} \right] \quad (49)$$

Inserting (48) into (49) we obtain the optimal rule:

$$\frac{1}{1 + i_t} = \min \left[1, \frac{(1 + \alpha_a) Z_t}{Z_a} (1 - N_t) + 1, \frac{1}{N_t} \right] \quad (50)$$

It is easy to check, using the fact that $N_t > Z_a / (1 + \alpha_a) Z_t$ (equation 47), that the third term is always smaller than the minimum of the two first

ones, so that we can suppress it. Subtracting from 1 we obtain the optimal rule (46). Again because knowledge of N_t and Z_t allows to recoup exactly the value of α_t (equation 48), rule (46) will allow to reach the same utility as the optimal rule in proposition 4. Q.E.D.

In order to interpret more easily rule (46), let us rewrite it as a function of Y_t and Z_t :

$$\frac{i_t}{1+i_t} = \max \left[0, \frac{1+\alpha_a}{Z_a} (Y_t - Z_t) \right] \quad (51)$$

Under preset prices a large value of Z_t creates a negative shock on employment, so it is natural to lower interest rates in such a situation. Now, for given Z_t , a high Y_t signals a positive demand shock, and therefore calls for an increase in the interest rate.

References

- [1] Abel, Andrew B. (1987), “Optimal monetary growth”, *Journal of Monetary Economics*, vol. 19, pp 437-450.
- [2] Bénassy, Jean-Pascal (2002), *The Macroeconomics of Imperfect Competition and Nonclearing Markets: A Dynamic General Equilibrium Approach*, Cambridge, Massachusetts: M.I.T. Press.
- [3] Bénassy, Jean-Pascal (2003), “Interest rate rules, inflation and the Taylor principle: an analytical exploration”, Cepremap, Paris, forthcoming, *Economic Theory*.
- [4] Friedman, Milton (1969), “The optimum quantity of money”, in M. Friedman (ed), *The Optimum Quantity of Money and Other Essays*. London: Macmillan.
- [5] Gray, Jo-Anna (1976), “Wage indexation: a macroeconomic approach”, *Journal of Monetary Economics*, vol. 2, pp 221-235.
- [6] Samuelson, Paul A. (1958), “An exact consumption-loan model of interest with or without the social contrivance of money”, *Journal of Political Economy*, vol. 66, pp 467-482.
- [7] Samuelson, Paul A. (1967), “A turnpike refutation of the golden rule in a welfare-maximizing many-year plan”, in K. Shell (ed), *Essays on the Theory of Optimal Economic Growth*, Cambridge: M.I.T. Press.
- [8] Samuelson, Paul A. (1968), “The two-part golden rule deduced as the asymptotic turnpike of catenary motions”, *Western Economic Journal*, vol. 6, pp 85-89.
- [9] Taylor, John B. (1993), “Discretion versus policy rules in practice”, *Carnegie-Rochester Series on Public Policy*, vol 39, pp 195-214.
- [10] Taylor, John B. (1998), “Monetary policy and the long boom”, *Federal Reserve Bank of Saint-Louis Review*, vol 80, n° 6, pp 3-11.
- [11] Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton and Oxford: Princeton University Press.