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Non Clearing Markets in General Equilibrium*

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Abstract

In this article we study models with non clearing markets in a full general equilibrium framework. The theories we describe synthesize three major schools of thought: Walrasian, Keynesian and imperfect competition. This synthesis is notably achieved by introducing quantity signals in addition to price signals into the traditional general equilibrium model. This considerably enlarges the scope of traditional general equilibrium, allowing us not only to construct equilibria with various price rigidities, but also to endogenize prices in a decentralized imperfect competition framework.

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In this article we study how to model situations of non clearing markets in a full general equilibrium framework. As we shall see from the historical discussion at the end, the theories we shall obtain synthesize three major schools of thought: (a) the Walrasian school, as Walras was the first to study a full fledged general equilibrium system; (b) the Keynesian school, as Keynes emphasized the importance of quantity adjustments in reaching a macroeconomic equilibrium with at least one non clearing market (that is, the labor market); and (c) the imperfect competition one which endogenized prices through explicit price making by agents internal to the system.

This synthesis is notably achieved by introducing quantity signals into the traditional general equilibrium model. These quantity signals are quantity constraints which tell each agent the maximum quantity he can trade in each market. As we shall see, the introduction of these quantity signals in addition to price signals considerably enlarges the scope of traditional general equilibrium since they allow to treat not only equilibria with various price rigidities, but also to endogenize prices in a decentralized imperfect competition framework.

The plan of the entry is the following: In the next three sections we shall describe the general concepts. The fourth section will give a brief historical outline of this line of thought.

1 Non clearing markets and quantity signals

In this section and the next two we describe various concepts in the framework of a monetary exchange economy where one good, money, serves as numéraire, medium of exchange and reserve of value (similar concepts have been developed for barter economies, see Bénassy, 1975b, 1982, but the formalization gets quite clumsy). There are ℓ markets in the period considered, where nonmonetary goods indexed by $h = 1, ..., \ell$ are exchanged against money at the price p_h. We call p the vector of these prices.

Agents are indexed by $i = 1, ..., n$. In market h agent i may make a purchase $d_{ih} \geq 0$ or a sale $s_{ih} \geq 0$. Define his net transaction of good h, $z_{ih} = d_{ih} - s_{ih}$, and z_i the ℓ-dimensional vector of these net transactions.

At the beginning of the period agent i holds quantities $\overline{m}_i$ of money, and ω_{ih} of good h. Call ω_i the vector of the ω_{ih}. As a result of his trades z_i, agent i ends up with final holdings of nonmonetary goods and money, x_i and m_i, given respectively by:

$$x_i = \omega_i + z_i \qquad m_i = \overline{m}_i - pz_i$$

We shall assume that agent i has a utility function on these final holdings $U_i(x_i, m_i) = U_i(\omega_i + z_i, m_i)$, which we assume throughout strictly concave in its arguments.

Walrasian equilibrium

In order to contrast it with the non Walrasian equilibrium concepts that will follow, let us describe briefly the Walrasian equilibrium of this economy (Arrow-Debreu, 1954, Debreu, 1959). Each agent i receives (from the implicit auctioneer) a price signal p. As a response he expresses a Walrasian net demand given by the function $z_i(p)$, solution in z_i of the following program:

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$

$$pz_i + m_i = \bar{m}_i$$

A Walrasian equilibrium price vector p^* is defined by the condition that all markets clear, i.e.:

$$\sum_{i=1}^{n} z_i(p^*) = 0$$

The vector of transactions realized by each agent i is $z_i(p^*)$.

Demands and transactions

As we will be studying non clearing markets, we must now make an important distinction, that between demands an supplies on the one hand, and the resulting transactions on the other.

Transactions, i.e. purchases or sales of goods, denoted d_{ih}^* and s_{ih}^*, are exchanges actually made, and must thus identically balance on each market, i.e.:

$$D_h^* = \sum_{i=1}^{n} d_{ih}^* = \sum_{i=1}^{n} s_{ih}^* = S_h^* \qquad \text{for all } h \tag{1}$$

On the other hand demands and supplies, denoted $\widetilde{d}_{ih}$ and $\widetilde{s}_{ih}$, are signals transmitted to the market (i.e. to the other agents) before exchange takes place. They represent as a first approximation the exchanges the agents wish to make on each market. So they do not necessarily match in a specific market, and no identity like (1) applies to them:

$$\widetilde{D}_h = \sum_{i=1}^{n} \widetilde{d}_{ih} \neq \sum_{i=1}^{n} \widetilde{s}_{ih} = \widetilde{S}_h$$

In order to shorten notation, we shall often work in what follows with net demands and net transactions defined respectively by:

$$\widetilde{z}_{ih} = \widetilde{d}_{ih} - \widetilde{s}_{ih} \qquad z^*_{ih} = d^*_{ih} - s^*_{ih}$$

The equality of aggregate purchases and sales (equation 1) is then rewritten:

$$\sum_{i=1}^{n} z^*_{ih} = 0 \qquad \text{for all } h \tag{2}$$

Rationing schemes

In each market h the exchange process must generate consistent transactions (i.e. transactions satisfying equations 1 or 2) from any set of possibly inconsistent demands and supplies. Some rationing will necessarily occur, which may take various forms, such as uniform rationing, queueing, priority systems, proportional rationing, etc ... depending on the particular organization of each market. We call *rationing scheme* the mathematical representation of each specific organization. To be more precise, the rationing scheme in market h is defined by a set of n functions:

$$z^*_{ih} = F_{ih}(\widetilde{z}_{1h}, ..., \widetilde{z}_{nh}) \qquad i = 1, ..., n \tag{3}$$

such that :

$$\sum_{i=1}^{n} F_{ih}(\widetilde{z}_{1h}, ..., \widetilde{z}_{nh}) = 0 \qquad \text{for all } \widetilde{z}_{1h}, ..., \widetilde{z}_{nh}$$

We assume that F_{ih} is continuous, nondecreasing in $\widetilde{z}_{ih}$ and nonincreasing in the other arguments. Before examining the possible properties of these rationing schemes, let us take a most simple example with two agents. Agent 1 emits a demand $\widetilde{d}_{1h}$, agent 2 a supply $\widetilde{s}_{2h}$. Then a natural rationing scheme, implicit in most macroeconomic models, is to take the level of transactions as equal to the minimum of demand and supply, i.e.:

$$d_{1h}^* = s_{2h}^* = \min(\widetilde{d}_{1h}, \widetilde{s}_{2h}) \tag{4}$$

Properties of rationing schemes

We first study two possible properties that a rationing scheme may satisfy: voluntary exchange and market efficiency.

The first property is actually an extremely natural one in a free market economy: We shall say that there is *voluntary exchange* in market h if no agent can be forced to purchase more than he demands, or to sell more than he supplies, which is expressed by:

$$d_{ih}^* \leq \widetilde{d}_{ih} \qquad s_{ih}^* \leq \widetilde{s}_{ih} \qquad \text{for all } i$$

or equivalently in algebraic terms:

$$|z_{ih}^*| < |\widetilde{z}_{ih}| \qquad z_{ih}^* \cdot \widetilde{z}_{ih} \geq 0 \qquad \text{for all } i.$$

Most markets in reality meet this condition, and we shall henceforth assume that it always holds. This allows us to classify agents in a market h in two categories: unrationed agents for which $z_{ih}^* = \widetilde{z}_{ih}$, and rationed agents who trade less than they wanted.

The second property we study here is that of market efficiency, or absence of frictions, which corresponds to the idea of exhaustion of all mutually advantageous exchanges: a rationing scheme on a market h is *efficient,* or *frictionless*, if one cannot find simultaneously a rationed demander and a rationed supplier in market h. The intuitive idea behind this is that in an efficiently organized market a rationed buyer and a rationed seller would meet and exchange until one of the two is not rationed. Together with voluntary exchange, it implies the "short-side rule", according to which agents on the "short-side" of the market can realize their desired transactions:

$$\widetilde{D}_h \geq \widetilde{S}_h \Rightarrow s_{ih}^* = \widetilde{s}_{ih} \qquad \text{for all } i$$

$$\widetilde{S}_h \geq \widetilde{D}_h \Rightarrow d_{ih}^* = \widetilde{d}_{ih} \qquad \text{for all } i$$

This rule also implies that the global level of transactions on a market h will be equal to the minimum of aggregate demand and supply:

$$D_h^* = S_h^* = \min\ (\widetilde{D}_h, \widetilde{S}_h)$$

We should note that the market efficiency assumption may not always hold, notably if one considers a fairly wide and decentralized market, because some demanders and suppliers might not meet pairwise. In particular the market efficiency property is usually lost by aggregation of submarkets (whereas the voluntary exchange property remains intact in the aggregation process). So we must keep in mind that it does not always hold. Fortunately this hypothesis is not necessary for most of the microeconomic concepts presented in the next sections.

Quantity signals

Now it is clear that at least rationed agents must perceive a quantity constraint in addition to the price signal. As it turns out, these quantity signals appear quite naturally in the formulation of a number of rationing schemes called *nonmanipulable*, which can be written under the form :

$$d_{ih}^* = \min(\widetilde{d}_{ih}, \overline{d}_{ih}) \qquad s_{ih}^* = \min(\widetilde{s}_{ih}, \overline{s}_{ih}) \tag{5}$$

where the quantity signals $\overline{d}_{ih}$ and $\overline{s}_{ih}$ are functions only of the demands and supplies of the other agents. As an example, we can note that the rationing scheme corresponding to equation (4) above is of this type with :

$$\overline{d}_{1h} = \widetilde{s}_{2h} \qquad \overline{s}_{2h} = \widetilde{d}_{1h}$$

For nonmanipulable schemes the relation between z_{ih}^* and $\widetilde{z}_{ih}$ looks as in figure 1, in which we see where the term nonmanipulable comes from: once rationed, the agent cannot increase, or "manipulate", the level of his transactions by increasing his demand and supply.

Figure 1

To make things a little more precise, let us rewrite the rationing scheme in market h (equation 3) under the form:

$$z_{ih}^* = F_{ih}(\widetilde{z}_{ih}, \widetilde{z}_{-ih}) \tag{6}$$

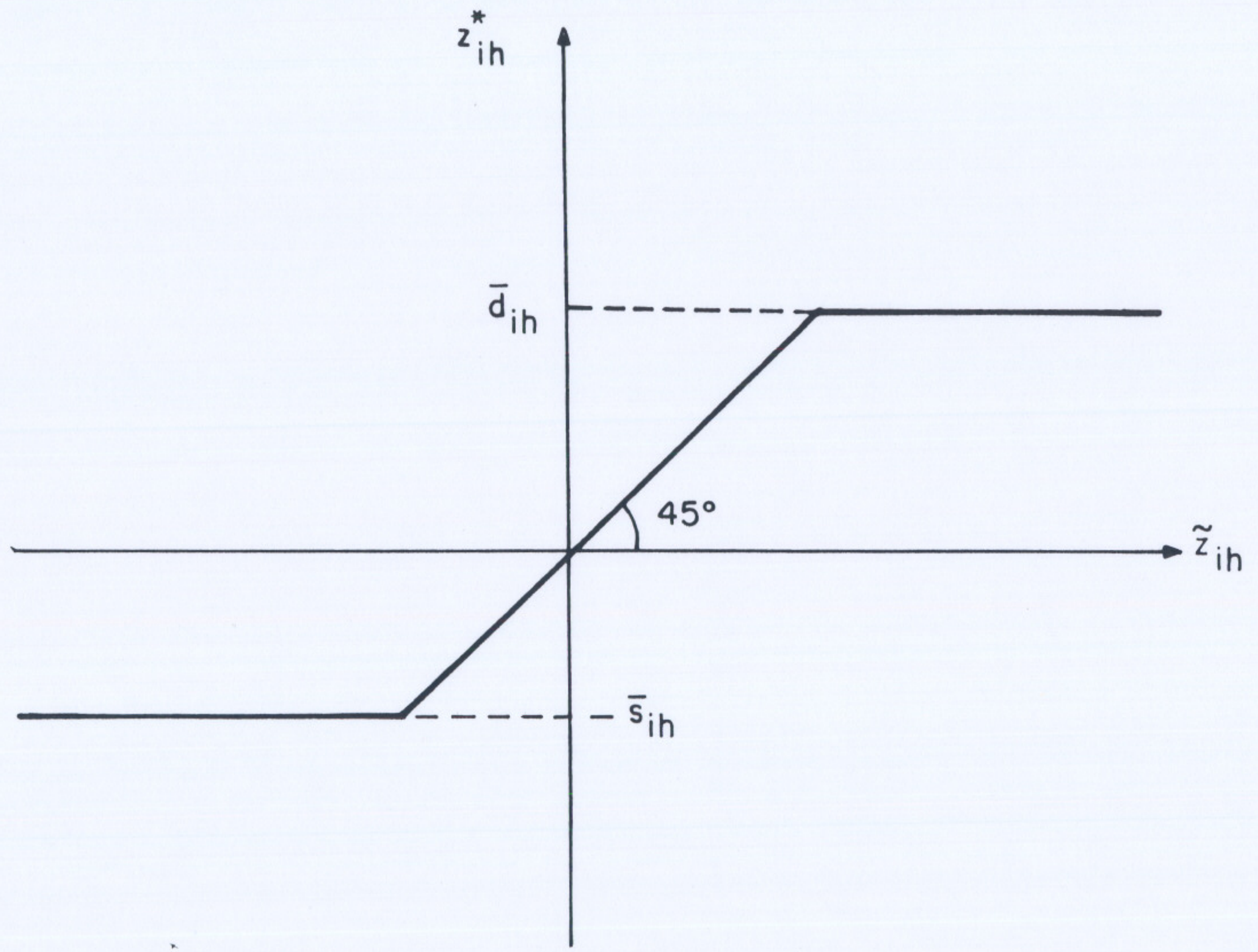

FIGURE 1

where $\widetilde{z}_{-ih}$ is the set of all net demands on market h, except that of agent i, i.e. $\widetilde{z}_{-ih} = \{\widetilde{z}_{jh} | j \neq i\}$. The rationing scheme is nonmanipulable if it can be rewritten as in equations (5), or algebraically:

$$F_{ih}(\widetilde{z}_{ih}, \widetilde{z}_{-ih}) = \begin{cases} \min(\widetilde{z}_{ih}, \overline{d}_{ih}) & \widetilde{z}_{ih} \geq 0 \\ \max(\widetilde{z}_{ih}, -\overline{s}_{ih}) & \widetilde{z}_{ih} \geq 0 \end{cases}$$

where $\overline{d}_{ih}$ and $\overline{s}_{ih}$ are functions of all demands and supplies in market h, except that of agent i, which we shall write as :

$$\overline{d}_{ih} = G^d_{ih}(\widetilde{z}_{-ih}) \geq 0 \qquad \overline{s}_{ih} = G^s_{ih}(\widetilde{z}_{-ih}) \geq 0 \tag{7}$$

Note that the functions $G^d_{ih}(\widetilde{z}_{-ih})$ and $G^s_{ih}(\widetilde{z}_{-ih})$ are not arbitrary, but are related to the rationing scheme F_{ih} through:

$$G^d_{ih}(\widetilde{z}_{-ih}) = \max \{\widetilde{z}_{ih} \mid F_{ih}(\widetilde{z}_{ih}, \widetilde{z}_{-ih}) = \widetilde{z}_{ih}\} \tag{8}$$

$$G^s_{ih}(\widetilde{z}_{-ih}) = -\min \{\widetilde{z}_{ih} \mid F_{ih}(\widetilde{z}_{ih}, \widetilde{z}_{-ih}) = \widetilde{z}_{ih}\} \tag{9}$$

where it appears clearly that these quantity constraints are indeed the maximum purchase and sale that agent i can make in market h.

We may note that some rationing schemes, called *manipulable*, such as the proportional rationing scheme, cannot be written under this form. The phenomenon of manipulation through demand and supply leads then to a perverse phenomenon of overbidding, and to the nonexistence of an equilibrium unless additional constraints are put on demands and supplies (Bénassy 1977b, 1982).

Most rationing schemes in the real world are actually nonmanipulable through demand and supply, and we shall thus from now on study only such rationing schemes as can be characterized by equations (5) or (7). The variables $\overline{d}_{ih}$ and $\overline{s}_{ih}$ in (5) and (7) are *quantity constraints.* These are the quantity signals that each agent receives, and they play a fundamental role in both quantity and price determination, as we will see in the next two sections. Before moving to the study of these problems and to the definition of non Walrasian equilibria, it is useful to rewrite equations (6) and (7) pertaining to an agent i under vector form :

$$z^*_i = F_i(\widetilde{z}_i, \widetilde{z}_{-i}) \qquad \overline{d}_i = G^d_i(\widetilde{z}_{-i}) \qquad \overline{s}_i = G^s_i(\widetilde{z}_{-i}) \tag{10}$$

where $\widetilde{z}_i$ is the vector of $\widetilde{z}_{ih}$, $h = 1, ..., \ell$, and $\widetilde{z}_{-i}$ is the set of all such vectors, except that of agent i himself, i.e. $\widetilde{z}_{-i} = \{\widetilde{z}_j | j \neq i\}$.

2 Fixprice equilibria

We now study a first concept of non Walrasian equilibrium, that of fixprice equilibrium. This concept is of interest for several reasons: First it will give us a very large class of consistent market allocations, since we shall find that under very standard conditions a fixprice equilibrium exists for every positive price system and every set of rationing schemes (we may note that Walrasian allocations are particular fixprice allocations, specifically those corresponding to a Walrasian price vector). Second, as we see in the next section, fixprice equilibria are a very important building block in constructing other non Walrasian equilibrium concepts with flexible prices.

We shall thus assume that the price system p is given. As we indicated we assume that the rationing schemes in all markets are nonmanipulable. Accordingly transactions and quantity signals are generated in all markets according to the formulas seen above (equations 8). We immediately see that all that remains to be done in order to obtain a fixprice equilibrium concept is to determine how demands themselves are formed, a task to which we now turn.

Effective demands and supplies

Demands and supplies are signals that agents send to the "market" (i.e. to the other agents) in order to obtain the best transactions. Consider thus an agent i faced with a price vector p and vectors of quantity constraints, $\overline{d}_i$ and $\overline{s}_i$. He knows that his transactions will be related to his demands and supplies by formulas (5) seen above, namely,

$$d_{ih}^* = \min(\widetilde{d}_{ih}, \overline{d}_{ih}) \qquad s_{ih}^* = \min(\widetilde{s}_{ih}, \overline{s}_{ih})$$

Now the problem is to choose a vector of net effective demands $\widetilde{z}_i$ which will lead him to the best possible transactions. As it turns out, there exists a simple and workable definition which generalizes Clower's (1965) original "dual decision" method: the effective demand of agent i on market h is the trade which maximizes his utility subject to the budget constraints and to the quantity constraints on the *other* markets. Formally the effective demand $\widetilde{z}_{ih}$ is solution in z_{ih} of the following programme :

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$
$$\begin{cases} pz_i + m_i = \overline{m}_i \\ -\overline{s}_{ik} \leq z_{ik} \leq \overline{d}_{ik} \qquad k \neq h \end{cases}$$

Because of the strict concavity of U_i, we obtain a function, denoted $\widetilde{\xi}_{ih}(p, \overline{d}_i, \overline{s}_i)$. Repeating the operation for all markets $h = 1, ...\ell$, we obtain a vector function of effective demands $\widetilde{\xi}_i(p, \overline{d}_i, \overline{s}_i)$. This vector of effective demands has two good properties: First, it leads to the best transactions that it is possible to attain given the price vector p and the quantity constraints $\overline{d}_i$ and $\overline{s}_i$. Secondly, whenever a constraint is binding on a market h, the corresponding demand or supply is greater than the quantity constraint, which thus "signals" to the market that the agent trades less than he would want. Such signals are useful to avoid trivial equilibria where no one would trade because nobody else signals that he wants to trade.

Fixprice equilibrium

With the above definition of effective demand, we are now ready to give a first definition of a fixprice equilibrium, found in Bénassy (1975a, 1982).

Definition 1 *A fixprice equilibrium associated to a price system p and rationing schemes represented by functions F_i, $i = 1, ..., n$, is a set of effective demands $\widetilde{z}_i$, transactions z_i^* and quantity constraints $\overline{d}_i$ and $\overline{s}_i$ such that:*

$$
\begin{array}{llll}
(a) & \widetilde{z}_i = \widetilde{\xi}_i(p, \overline{d}_i, \overline{s}_i) & & i = 1, ..., n \\
(b) & z_i^* = F_i(\widetilde{z}_i, \widetilde{z}_{-i}) & & i = 1, ..., n \\
(c) & \overline{d}_i = G_i^d(\widetilde{z}_{-i}) & \overline{s}_i = G_i^s(\widetilde{z}_{-i}) & i = 1, ..., n
\end{array}
$$

Equilibria defined in this way exist for all positive prices and all rationing schemes satisfying voluntary exchange and nonmanipulability (Bénassy 1975a, 1982). The "exogenous" data are the price system p and the rationing schemes F_i, $i = 1, ...n$. One may wonder whether for given such exogenous data the equilibrium is likely to be unique. A positive answer has been given by Schulz (1983) who showed that the equilibrium is globally unique, provided the "spillover" effects (there is a spillover effect when a binding constraint in one market modifies the effective demand in an other market) are less than one hundred percent in value terms. For example in the simplest Keynesian model this would amount to a propensity to consume strictly smaller than 1.

In what follows we will assume that the Schulz conditions hold, and denote by $\widetilde{Z}_i(p)$, $Z_i^*(p)$, $\overline{D}_i(p)$ and $\overline{S}_i(p)$ the functions giving the values of $\widetilde{z}_i$, z_i^*, $\overline{d}_i$

and $\overline{s}_i$ at a fixprice equilibrium corresponding to p (the market organization, and thus the rationing schemes, being assumed invariant).

An alternative concept

We shall now present an alternative concept of fixprice equilibrium, due to Drèze (1975) (who actually dealt with the more general case of prices variable between fixed limits), and which we shall recast using our notations. That concept does not separate demands from transactions, and thus considers directly the vectors of transactions z_i^* and quantity constraints $\overline{d}_i$ and $\overline{s}_i$. The original concept actually assumed uniform rationing, so that the vectors $\overline{d}_i$ and $\overline{s}_i$ were the same for all agents.

Definition 2 *A fixprice equilibrium for a given set of prices p is defined as a set of transactions z_i^* and quantity constraints $\overline{d}_i$ and $\overline{s}_i$ such that:*

(a) $\quad \sum_{i=1}^{n} z_{ih}^* = 0 \qquad \forall\ h$

(b) $\quad$ *The vector z_i^* is solution in z_i of :*

$$\textit{Maximize } U_i(\omega_i + z_i, m_i) \qquad s.t.$$
$$\begin{cases} pz_i + m_i = \overline{m}_i \\ -\overline{s}_{ih} \leq z_{ih} \leq \overline{d}_{ih} \qquad \forall\ h \end{cases}$$

(c) $\forall\ h$ $z_{ih}^* = \overline{d}_{ih}$ *for some i implies* $z_{jh}^* > -\overline{s}_{jh} \qquad \forall\ j$
$\quad z_{ih}^* = -\ \overline{s}_{ih}$ *for some i implies* $z_{jh}^* < -d_{jh} \quad \forall\ j$

Let us now interpret these conditions. Condition (a) is the natural requirement that transactions should balance in each market. Condition (b) says that transactions must be individually rational, i.e. they must maximize utility subject to the budget constraint and the quantity constraints on all markets. We may note at this stage that using quantity constraints under the form of upper and lower bounds on trades implicitly assumes rationing schemes which exhibit voluntary exchange and nonmanipulability, as we saw when studying rationing schemes. Condition (c) says that rationing may affect either supply or demand, but not both simultaneously. We recognize here with a different formalization the condition of market efficiency which is thus built into this definition of equilibrium, whereas it is not in the previous definition.

Drèze (1975) proved that an equilibrium according to definition 2 exists for all positive price systems and for uniform rationing schemes under the standard concavity assumptions for the utility functions. The concept is easily extended to nonuniform bounds (Grandmont-Laroque, 1976, Greenberg-Müller, 1979), but in this last case it is not specified in the concept how shortages are allocated. Because of this there will be usually an infinity of equilibria corresponding to a given price vector, as soon as there are two rationed agents on one side of a market.

As we noted above, the two concepts of fixprice equilibrium we described in this section are based, implicitly or explicitly, on a representation of markets under the form of rationing schemes satisfying voluntary exchange and nonmanipulability. This suggests that if in the first definition we further assume that all rationing schemes are efficient or frictionless the two definitions should yield similar sets of equilibrium allocations for a given price system. This was indeed proved by Silvestre (1982, 1983) for both exchange and production economies. The relation between the two concepts have been further explored by D'Autume (1985).

3 Price making and equilibrium

As this stage we still need a description of price making by agents internal to the system. We shall describe in this section a concept dealing with that problem and we shall see that, just as in demand and supply theory, quantity signals play a prominent role. It is indeed quite intuitive that quantity constraints must be a fundamental part of the competitive process in a decentralized economy: It is the inability to sell as much that they want which leads suppliers to propose, or accept from other agents, a lower price, and conversely it is the inability to purchase as much as they want that leads demanders to propose, or accept, a higher price.

Various modes of price making integrating these aspects can be envisioned. We deal here with a realistic organization of the pricing process where agents on one side of the market (most often the suppliers) quote prices and agents on the other side act as price takers. The general idea relating the concepts in this section to those of the previous one is that price makers change their prices so as to "manipulate" the quantity constraints they face (that is, so as to increase or decrease their possible sales or purchases). As we shall see, this model of price making is quite reminiscent of the imperfect competition line: Chamberlin (1933), Robinson (1933), Triffin (1940), Bushaw-Clower (1957), Arrow (1959), and more particularly of the theories of general equilibrium with monopolistic competition, as developed notably by Negishi (1961).

The framework

We thus now assume that agent i controls the prices of a possibly empty) subset H_i of goods. Goods are distinguished both by their physical characteristics and by the agent who sets their price. We thus consider two goods sold by different sellers as different goods, a fairly natural assumption since these goods differ at least by location, quality, etc..., so that:

$$H_i \cap H_j = \{\phi\} \qquad i \neq j$$

We shall denote by p_i the set of prices controlled by agent i and p_{-i} the rest of prices, i.e.:

$$p_i = \{p_h | h \in H_i\} \qquad p_{-i} = \{p_h | h \notin H_i\}$$

Each agent chooses his price vector p_i taking the other prices p_{-i} as given. The equilibrium structure is thus that of a Nash-equilibrium in prices, corresponding to an idea close to that of monopolistic competition. The basic idea behind the modelling of price making itself in such models is, as we indicated above, that each price maker uses the prices he controls to "manipulate" the quantity constraints he faces. Consider the markets whose price are determined by agent i, and subdivide further H_i into H_i^d (goods demanded by i) and H_i^s (goods supplied by i). We may note in passing that, although agent i appears formally as a monopolist in markets $h \in H_i^s$ or a monopsonist in markets $h \in H_i^d$, his actual "monopoly power" may be very low due to the fact that other agents sell or buy products which are extremely close substitutes to those he controls. Because the price makers are alone on their side of the markets where they set prices, their quantity constraints on these markets have the simple form:

$$\overline{s}_{ih} = \sum_{j \neq i} \widetilde{d}_{jh} \qquad h \in H_i^s$$

$$\overline{d}_{ih} = \sum_{j \neq i} \widetilde{s}_{jh} \qquad h \in H_i^d$$

i.e. the maximum quantity that price setter i can sell is the total demand of the others, and conversely if he is a buyer. All we need to know, in order to pose the problem of price setting as a standard decision problem, is the relation, as perceived by the price maker, between the quantity constraints he faces and the prices he sets. Several approaches allow to treat this problem and to link it with the concepts seen previously. The first one, based on Negishi's (1961) subjective demande curve approach, was developed in Benassy (1976, 1982). The second one is an objective demand curve approach, developed in Benassy (1987, 1988), and which we shall now briefly describe.

Objective demand curves

The implicit idea behind the objective demand curve approach (Gabszewicz and Vial, 1972, Marschak and Selten, 1976, Nikaido, 1975) is that each price maker knows the economy well enough to be able to compute under all circumstances the actual quantity constraints he will face. Since we are considering a Nash equilibrium, he must be able to perform this computation for any set p_i of prices he chooses as well as for any set p_{-i} of the other prices, that is, he must be able to compute his constraints for any vector of prices, once all feedback effects have been accounted for.

But we know from the previous section that, for a given organization of the economy (i.e. notably for given rationing schemes), and for a given set of prices p, the quantity constraints agent i faces are given by the functions $\overline{D}_i(p)$ and $\overline{S}_i(p)$. If the agent has full knowledge of the parameters of the economy (a strong assumption, of course, but which is embedded in the notion of an objective demand curve), then he knows this and the objective demand and supply curves will be respectively given by the functions $\overline{S}_i(p)$ and $\overline{D}_i(p)$. We may note that the objective demand curve $\overline{S}_i(p)$ is denoted as a constraint on agent $i's$ supply, which is natural since the sum of all other agents' demands acts as a constraint on the sales of agent i, and symmetrically with the objective supply curve $\overline{D}_i(p)$.

Price making and equilibrium

If agent i knows the two vector functions $\overline{D}_i(p)$ and $\overline{S}_i(p)$, the programme giving his optimal price p_i is the following :

$$\text{Maximize } U_i(\omega_i + z_i, m_i) \qquad \text{s.t.}$$
$$\begin{cases} pz_i + m_i = \overline{m}_i \\ -\overline{S}_i(p) \leq z_i \leq \overline{D}_i(p) \end{cases}$$

which yields the optimum price p_i chosen by agent i as a function of the other prices p_{-i} :

$$p_i = \psi_i(p_{-i})$$

This naturally leads us to the definition of an equilibrium with price makers:

Definition 3 *An equilibrium with price makers is characterized by a set of prices p_i^*, net demands $\widetilde{z}_i$, transactions z_i^* and quantity constraints $\overline{d}_i$ and $\overline{s}_i$ such that :*
(a) $\quad p_i^* = \psi_i(p_{-i}^*)$
(b) $\quad \widetilde{z}_i$, z_i^*, $\overline{d}_i$, $\overline{s}_i$ *are equal respectively to* $\widetilde{Z}_i(p^*)$, $Z_i^*(p^*)$, $\overline{D}_i(p^*)$, $\overline{S}_i(p^*)$.

Condition (a) indicates that we have a Nash equilibrium in prices, given each agent's optimal price responses. Condition (b) says that the various quantities form a fixprice equilibrium (according to definition 1) for the price vector p^*. Further discussion and conditions for existence can be found in Bénassy (1988, 1990).

4 Bibliographical references

So far we have concentrated in this entry on the microeconomic concepts allowing us to deal with non clearing markets at a general equilibrium level. We shall now indicate further bibliographical references both on the early history of the domain and on macroeconomic applications.

History

The field we described in this entry has a triple ancestry: On one hand Walras (1874) developed a model of general equilibrium with interdependent markets where adjustment was made through prices. This model, in its modern reformulation (Arrow-Debreu, 1954, Arrow, 1963, Debreu, 1959) has become the basic benchmark concept in microeconomics. On the other hand Keynes (1936) and Hicks (1937) built, at the macroeconomic level, a concept of equilibrium where adjustment was made by quantities (the level of national income) as well as by prices. Finally, following the contributions by Chamberlin (1933) and Robinson (1933), progress was made on the treatment of imperfect competition. Notably Negishi (1961) formalized imperfect competition with subjective demand curves in a general equilibrium framework.

A few isolated contributions in the postwar period made some steps towards modern theories of non clearing markets: Bent Hansen (1951) introduced the ideas of active demand, close in spirit to that of effective demand, and of quasi-equilibrium where persistent disequilibrium created steady inflation. Patinkin (1956, ch. 13) considered the situation where firms might not be able to sell all their Walrasian output. Hahn and Negishi (1962) studied non-tatonnement processes where trade could take place before a general equilibrium price system was reached.

A stimulating impetus came from the contributions of Clower (1965) and Leijonhufvud (1968), who reinterpreted Keynesian analysis in terms of market rationing and quantity adjustments. These insights were included in the first fixprice-fixwage macroeconomic model by Barro and Grossman (1971, 1976).

The main subsequent development was the construction of rigorous microeconomic concepts allowing us to deal with non clearing markets and imperfect competition in a full multimarket general equilibrium setting, as we described above. Notably Drèze (1975) and Bénassy (1975a, 1977b, 1982) bridged the gap between the Walrasian and Keynesian lines of thought by generalizing the Walrasian equilibrium concept to integrate non clearing markets and quantity signals. The link between this new line of work and the imperfect competition equilibrium concepts in the Negishi (1961) line was made in Bénassy (1976, 1977a, 1988). These contributions led to the unified framework we set out described in the previous sections. Of course, since one of the main goals of this line of research was to bridge the gap between microeconomics and macroeconomics, there were a number of macroeconomic applications of the above concepts, which we now briefly describe.

Macroeconomic applications

As indicated above, the first fully worked out fixprice-fixwage macroeconomic model imbedding the notions set out above is that of Barro-Grossman

(1971, 1976). Early attempts are found in Glustoff (1968) and Solow and Stiglitz (1968). Further developments of the model were made in Bénassy (1977a, 1982, 1986), Malinvaud (1977), Hildenbrand and Hildenbrand (1978), Muellbauer and Portes (1978), Honkapohja (1979), Neary and Stiglitz (1983), Persson and Svensson (1983). Most of these models concentrated on the problem of employment and policy. Other problems have been treated with this methodology, including notably foreign trade (Dixit, 1978, Neary, 1980, Cuddington, Johansson and Löfgren, 1984), Growth (Ito, 1980, Picard, 1983, D'Autume, 1985), Business cycles (Bénassy, 1984), as well as the specific problems of planned socialist economies (Portes, 1981).

An important part of this line of macroeconomic modelling is that concerned with the explicit introduction of price making and imperfect competition in the macrosetting. Models of that type can be found notably in Bénassy (1977a, 1982, 1987, 1990, 1991), Negishi (1977, 1979), Hart (1982), Snower (1983) Weitzman (1985), Svensson (1986), Blanchard and Kiyotaki (1987), Dixon (1987), Sneessens (1987), Silvestre (1988) and Jacobsen and Schultz (1990).

Now the concepts described in this entry are full general equilibrium models in the tradition of, say, Arrow and Debreu (1954) and Debreu (1959). Contemporaneously to these developments other authors developed, under the initial name of real business cycles, dynamic stochastic models based on the hypothesis of rational expectations. At some point these two lines of work were synthesized, and the result of this synthesis is described in the dictionary article "Dynamic models with non clearing markets".

References

[1] Arrow, Kenneth J. (1959) "Towards a theory of price adjustment". In *The Allocation of Economic Resources*, ed. M. Abramowitz. Stanford: Stanford University Press.

[2] Arrow, Kenneth J. (1963), "The role of securities in the optimal allocation of risk-bearing", *Review of Economic Studies*, vol. 31, pp 91-96.

[3] Arrow, Kenneth J. and Gérard Debreu (1954), "Existence of an equilibrium for a competitive economy", *Econometrica,* vol. 22, pp 265-290.

[4] Barro, Robert J. and Herschel I. Grossman (1971), "A general disequilibrium model of income and employment", *American Economic Review,* vol. 61, pp 82-93.

[5] Barro, Robert J. and Herschel I. Grossman (1976), *Money, Employment and Inflation.* Cambridge: Cambridge University Press.

[6] Bénassy, Jean-Pascal (1975a), "Neo-Keynesian disequilibrium theory in a monetary economy", *Review of Economic Studies,* vol. 42*,* pp 503-523.

[7] Bénassy, Jean-Pascal (1975b), "Disequilibrium exchange in barter and monetary economies", *Economic Inquiry,* vol. 13*,* pp 131-156.

[8] Bénassy, Jean-Pascal (1976), "The disequilibrium approach to monopolistic price setting and general monopolistic equilibrium", *Review of Economic Studies,* vol. 43*,* pp 69-81.

[9] Bénassy, Jean-Pascal (1977a), "A Neokeynesian model of price and quantity determination in disequilibrium". In *Equilibrium and Disequilibrium in Economic Theory*, ed G. Schwödiauer. Boston: Reidel Publishing Company.

[10] Bénassy, Jean-Pascal (1977b), "On quantity signals and the foundations of effective demand theory", *The Scandinavian Journal of Economics,* vol. 79, pp 147-168.

[11] Bénassy, Jean-Pascal (1982), *The Economics of Market Disequilibrium.* Academic Press, New York.

[12] Bénassy, Jean-Pascal (1984), "A non-Walrasian model of the business cycle", *Journal of Economic Behavior and Organization,* vol. 5, pp 77-89.

[13] Bénassy, Jean-Pascal (1986), *Macroeconomics: An Introduction to the Non-Walrasian Approach,* Orlando, Florida: Academic Press.

[14] Bénassy, Jean-Pascal (1987), "Imperfect competition, unemployment and policy", *European Economic Review,* vol. 31, pp 417-426.

[15] Bénassy, Jean-Pascal (1988), "The objective demand curve in general equilibrium with price makers", *The Economic Journal,* supplement, vol. 98, pp 37-49.

[16] Bénassy, Jean-Pascal (1990), "Non Walrasian equilibria, money and macroeconomics". In *Handbook of Monetary Economics*, ed. B. Friedman and F. H. Hahn. Amsterdam: North-Holland.

[17] Bénassy, Jean-Pascal (1991), "Microeconomic foundations and properties of a macroeconomic model with imperfect competition". In *Issues in Contemporary Economics, Volume 1: Markets and Welfare*, ed. K. J. Arrow. London: Macmillan.

[18] Blanchard, Olivier J. and Nobuhiro Kiyotaki (1987), "Monopolistic competition and the effects of aggregate demand" *American Economic Review,* vol. 77, pp 647-666.

[19] Bushaw, D.W., and Robert W. Clower (1957), *Introduction to Mathematical Economics.* Homewood, Illinois: Richard D. Irwin.

[20] Chamberlin, Edward H. (1933), *The Theory of Monopolistic Competition,* 7th edn. Cambridge, MA: Harvard University Press, 1956.

[21] Clower, Robert W., (1965), "The Keynesian counterrevolution: a theoretical appraisal". In *The Theory of Interest Rates*, ed. F. H. Hahn and F.P.R. Brechling. London: Macmillan.

[22] Cuddington, John T., Per-Olof Johansson and Karl Gustav Löfgren (1984), *Disequilibrium Macroeconomics in Open Economies,* Oxford: Basil Blackwell.

[23] D'Autume, Antoine (1985), *Monnaie, Croissance et Déséquilibre.* Paris: Economica.

[24] Debreu, Gérard (1959), *Theory of Value.* New York: Wiley.

[25] Dixit, Avinash K. (1978), "The balance of trade in a model of temporary equilibrium with rationing", *Review of Economic Studies,* vol. 45, pp 393-404.

[26] Dixon, Huw (1987), "A simple model of imperfect competition with Walrasian features", *Oxford Economic Papers,* vol. 39, pp 134-160.

[27] Drèze, Jacques H. (1975), "Existence of an exchange equilibrium under price rigidities", *International Economic Review,* vol. 16, pp 301-320.

[28] Gabszewicz, Jean Jaskold and Jean-Philippe Vial (1972), "Oligopoly "A la Cournot" in a general equilibrium analysis", *Journal of Economic Theory*, vol. 42, pp 381-400.

[29] Glustoff, E. (1968), "On the existence of a Keynesian equilibrium", *Review of Economic Studies,* vol. 35, pp 327-334.

[30] Grandmont, Jean-Michel, and Guy Laroque (1976), "On Keynesian temporary equilibria", *Review of Economic Studies,* vol. 43, pp 53-67.

[31] Greenberg, J. and H. Muller (1979), "Equilibria under price rigidities and externalities". In *Game Theory and Related Topics*, ed. O. Moeschlin and D. Pallaschke. Amsterdam: North-Holland.

[32] Hahn, Frank, H. and Takashi Negishi (1962), "A theorem on non tatonnement stability", *Econometrica,* vol. 30, pp 463-469.

[33] Hansen, Bent (1951), *A Study in the Theory of Inflation.* London: Allen and Unwin.

[34] Hart, Oliver D. (1982), "A model of imperfect competition with Keynesian features", *Quarterly Journal of Economics,* vol. 97, pp 109-138.

[35] Hicks, John, R. (1937), "Mr. Keynes and the classics: a suggested interpretation", *Econometrica,* vol. 5, pp 147-159.

[36] Hildenbrand, Kurt and Werner Hildenbrand (1978), "On Keynesian equilibria with unemployment and quantity rationing", *Journal of Economic Theory,* vol. 18, pp 255-277.

[37] Honkapohja, Seppo (1979), "On the dynamics of disequilibria in a macro model with flexible wages and prices". In *New Trends in Dynamic System Theory and Economics*, ed. M. Aoki and A. Marzollo. New York: Academic Press.

[38] Ito, Takatoshi (1980), "Disequilibrium growth theory", *Journal of Economic Theory,* vol. 23, pp 380-409.

[39] Jacobsen, Hans-Jörgen and Christian Schultz (1990), "A general equilibrium macro model with wage bargaining", *Scandinavian Journal of Economics,* vol. 92, pp 379-398.

[40] Keynes, John Maynard (1936), *The General Theory of Money, Interest and Employment,* New York: Harcourt Brace.

[41] Leijonhufvud, Axel (1968), *On Keynesian Economics and the Economics of Keynes.* Oxford: Oxford University Press.

[42] Malinvaud, Edmond (1977), *The Theory of Unemployment Reconsidered,* Oxford: Basil Blackwell.

[43] Marschak, Thomas and Reinhard Selten (1974), *General Equilibrium with Price-Making Firms.* Berlin: Springer-Verlag.

[44] Muellbauer, John, and Richard Portes (1978), "Macroeconomic models with quantity rationing", *Economic Journal,* vol. 88, pp 788-821.

[45] Neary, Peter J. (1980), "Nontraded goods and the balance of trade in a neo-Keynesian temporary equilibrium", *Quarterly Journal of Economics,* vol. 95, pp 403-430.

[46] Neary, Peter J., and Joseph E. Stiglitz (1983), "Towards a reconstruction of Keynesian economics: expectations and constrained equilibria", *Quarterly Journal of Economics,* vol. 98, supplement, pp 199-228.

[47] Negishi, Takashi (1961), "Monopolistic competition and general equilibrium", *Review of Economic Studies,* vol. 28, pp 196-201.

[48] Negishi, Takashi (1977), "Existence of an underemployment equilibrium". In *Equilibrium and Disequilibrium in Economic Theory*, ed. G. Schwödiauer. Boston: Reidel Publishing Company.

[49] Negishi, Takashi (1979), *Microeconomic Foundations of Keynesian Macroeconomics.* Amsterdam: North-Holland.

[50] Nikaido, Hukukane (1975), *Monopolistic Competition and Effective Demand.* Princeton: Princeton University Press.

[51] Patinkin, Don (1956), *Money, Interest and Prices*, second edition. New-York: Harper and Row, 1965.

[52] Persson, Torsten and Lars E.O. Svensson (1983), "Is optimism good in a Keynesian economy ?", *Economica,* vol. 50, pp 291-300.

[53] Picard, Pierre (1983), "Inflation and growth in a disequilibrium macroeconomic model", *Journal of Economic Theory,* vol. 30, pp 266-295.

[54] Portes, Richard (1981), "Macroeconomic equilibrium and disequilibrium in centrally planned economies", *Economic Inquiry,* vol. 19, pp 559-578.

[55] Robinson, Joan (1933), *The Economics of Imperfect Competition,* second edition. London: Macmillan, 1969.

[56] Schulz, Norbert (1983), "On the global uniqueness of fixprice equilibria", *Econometrica,* vol. 51, pp 47-68.

[57] Silvestre, Joaquim (1982), "Fixprice analysis of exchange economies", *Journal of Economic Theory,* vol. 26, pp 28-58.

[58] Silvestre, Joaquim (1983), "Fixprice analysis in productive economies", *Journal of Economic Theory,* vol. 30, pp 401-409.

[59] Silvestre, Joaquim (1988), "Undominated prices in the three good model", *European Economic Review,* vol. 32, pp 161-178.

[60] Sneessens, Henri (1987), "Investment and the inflation-unemployment trade off in a macroeconomic rationing model with monopolistic competition", *European Economic Review,* vol. 31, pp 781-815.

[61] Snower, Dennis (1983), "Imperfect competition, unemployment and crowding out", *Oxford Economic Papers,* vol. 35, pp 569-584.

[62] Solow, Robert M. and Joseph E. Stiglitz (1968), "Output, employment and wages in the short run", *Quarterly Journal of Economics,* vol. 82, pp 537-560.

[63] Svensson, Lars E.O. (1986), "Sticky goods prices, flexible asset prices, monopolistic competition and monetary policy", *Review of Economic Studies,* vol. 53, pp 385-405.

[64] Triffin, Robert (1940), *Monopolistic Competition and General Equilibrium Theory;* Cambridge, MA: Harvard University Press.

[65] Walras, Léon (1874), *Eléments d'économie politique pure.* Lausanne: Corbaz. Definitive edition translated by W. Jaffe, *Elements of Pure Economics.* London: Allen and Unwin, 1954.

[66] Weitzman, Martin L. (1985), "The simple macroeconomics of profit sharing", *American Economic Review,* vol. 75, pp 937-953.