Fuzzy Capital Requirements, Risk-Shifting and the Risk Taking Channel of Monetary Policy*†

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Abstract

We set up a model where asset price bubbles due to risk shifting can be moderated by capital requirements. However, imperfect information about the ratio of required capital, or, in the context of the sub-prime crisis, the extent of regulatory arbitrage, introduces uncertainty about the risk exposure of intermediaries. Underestimation of regulatory arbitrage may induce households to infer that higher asset prices are due to a decline of risk. First, this mechanism can explain why the risk premia paid by US financial intermediaries did not increase between 2000 and 2007 in spite of its increasing leverage. Second, we provide a theory of the risk taking channel of monetary policy: in the model, the underestimation of risk is larger the lower the level of the risk free interest rate

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1 Introduction

Many economists and institutions recognize that the wave of financial innovations that took place in the years 2000 got out of control.[1] The increase of financial intermediaries' leverage through various off-balance sheet innovations appear to have fed what will most likely enter history as another episode of asset price bubble.

For many observers,[2] the engine of this "bubble" was the creativity deployed by financial intermediaries to increase their return on capital through higher leverage. In the case of financial intermediaries subject to capital requirement, this creativity is usually labelled *regulatory arbitrage*. However, this increase in leverage, which is akin to being subject to a higher risk of default, was not sanctioned by higher risk premia on intermediaries' debts. These premia remained low and non-increasing until the summer of 2007.

The goal of this paper is to understand why financial intermediaries were able to pay non-increasing risk premia while increasing leverage and risk-taking, and to derive implications about the build-up of financial fragility. We also analyze whether the stance of monetary policy influence the perception of risk and the incentives of risk taking.

To do so, we build a model of asset pricing where intermediaries can default on their debt in case of bad aggregate outcomes. In this set-up we derive the interest rate margin[3] paid on the debt of intermediaries as a function of leverage in a simple general equilibrium setting.

Our contribution is two fold. First, the patterns of risk premia and leverage ratios observed in the US between 2000 and 2007 can be understood only if agents underestimated the extent of regulatory arbitrage and thus the intermediaries' incentives to take

[1] "Those of us who have looked to the self-interest of lending institutions to protect shareholders' equity, myself included, are in a state of shocked disbelief,"

Extract of Alan Greespan's 23 October 2008 testimony to the House Committee on Oversight and Government Reform.

[2] Brunnermeier (2008), Blanchard (2008), Greenlaw et al. (2008), Acharya et al. (2009).

[3] Through out the text we will call this interest rate margin a risk premium even if, in the model, agents are risk neutral. These agents require an interest rate margin that, ex ante, covers exactly the expected cost of default. Although they do not require a premium for the risk associated to this transaction, such a premium would only strengthen our argument.

risk. We show how rational investors may wrongly deduce from rising asset prices that the aggregate risk is decreasing, and thus charge a low risk premium on debt.

Second, we provide a theory for the risk taking channels of monetary policy. In particular, if uninformed investors underrate the extent of regulatory arbitrage, loose monetary policy may amplify their underestimation of risks. The underestimation of risk is larger the lower the level of interest rate. This is because lower interest rates imply a larger effect of capital requirements on the price of risky assets and in turn on the perception of risk by uninformed investors. In a sense this effect is no different from the well-know non linearity of asset price valuation formula. Lowering the discount rate from 3 to 2 % has a larger effect on stock prices than lowering it from 6 to 5 %.

This risk taking channel of monetary policy implies both mis-perception of risk by some investors and increased exposure to risks by others. It is however conditional on a lack of transparency in capital requirements, a feature of the years 2000 and, in all likelyhood, of any phase of major financial innovations or deregulation.

More specifically, we set up a model à la Allen and Gale (2000), which is enriched to analyze the role of regulatory capital. In this model, investors can invest in risky assets only indirectly, through lending to financial intermediaries. Investors require a risk premium on this loan because they anticipate that financial intermediaries will default in the bad state of the world. However, the limited liabilities of intermediaries in case of default imply that they take too much risk. A bubble results in the sense that the price of the risky asset is higher than in the case where households can invest directly in the risky asset. In this set-up, we introduce capital requirements as a constraint put on intermediaries. We assume that they have to invest some of their own resources to finance investments in the risky asset. This constraint hence moderates the degree of risk taking by intermediaries and the distortions induced by their limited liabilities. Although households do not know the risk of the risky asset ex ante, they may infer it from asset prices and from regulatory capital requirements.

We describe how investors infer the underlying risk of the risky asset for two assumptions on the information set of households. First, we assume that they know the exact amount of capital requirement imposed on intermediaries. We prove that they are then able to deduce the underlying risk of the risky asset from asset prices. In this case, an (anticipated) decrease in capital requirements raises both the price of the risky asset and the premium charged on intermediaries. This model therefore falls short of reproducing the build up of the subprime crisis during which the increased leverage of the commercial banks did not imply higher risk premia.

Second, we assume that investors do not know the exact severity of the regulatory constraint, and thus the degree of risk-shifting, and try to infer it from asset prices. One of the reasons why the exact amount of regulatory capital ratio can be opaque is, as argued by Acharya and Schnabl (2009), intermediaries use off balance sheets conduit to "play" the capital requirements. The uncertainty about the strength of the regulatory constraint is thus modeled as an uncertainty about the amount of risk associated to assets held by banks. In this case, the equilibrium is indexed on the beliefs of uninformed investors about the degree of intermediaries' capital requirement. We prove that households underestimate the riskiness of the asset if they overestimate capital requirements of intermediaries. The model can hence replicate one of the most puzzling stylized fact of the banking crisis. Risk premia did not increase because the depletion of capital that financial intermediaries effectively pledged to their riskiest investments was underestimated by uninformed investors (be they households, pension funds, regulators or even managers of the largest banks).

We then study the effect of monetary policy on risk perception and incentives in the context of our model. We find that the level of the riskless interest rate affects the signal extraction problem of households. In cases where they underestimate the degree of regulatory arbitrage, lower real interest rates lead them to underestimate risk, which in turn amplify the "bubble". In the final section of the paper we argue that this explanation of the build up of financial fragility seems consistent with the data.

Related literature.

The focus of our article is on the link between leverage, asset prices and regulation. Thus our paper is related to the results of Adrian and Shin (2008) and Geanakoplos (2009) who have highlighted theoretically and empirically the impact of the financial intermediaries' leverage on asset prices. It also provides a theoretical underpinning for the empirical results of Ioanidou, Ongena and Peydro (2008), Maddaloni, Peydró and Scopel (2008), Altunbas, Gambacorta and Marques-Ibanez (2009) and Adrian and Shin (2008) who showed that accommodative monetary policy stance are associated with more risk taking by banks. We hence provide a rational for what Borio and Zhu (2009) call the "risk-taking" channel of monetary policy.

In the risk-shifting literature, our paper relates to a number of recent papers. Allen and Gale (2000) seminal paper on risk-shifting showed how limited liabilities of debt issuers leads to over-investment in risky assets. This is because they care only about the up-side of the return distribution. Barlevy (2008) proved that risk-shifting also implies bubbles in more general frameworks of financial intermediation (i.e. when the formation of financial contracts is endogenous); He also generalized risk-shifting to a continuous time dynamic framework. Challe and Ragot (2008) expand the risk-shifting model to the case where the supply of loans is endogenous[4].

Our paper is also linked to the literature discussing the opacity around the real cost of risk-taking for financial intermediaries in terms of capital requirements. Asharya and Schnabl (2009) show how banks have been able to undervalue their "real" leverage ratio through off-balance sheet operations. Despite the transfer of risky assets to Special Purpose Vehicle, recent episodes demonstrate that risks were still on the book explicitely or implicitely, either because banks were tied by explicit liquidity and credit enhancement contracts or for reputational motives: ABS have frequently been brought back to intermediaries balance sheet after 2007, once in the bad state of the world. Then, the regulatory arbitrage added to the complexity of the capital requirements calculus (Rochet 2008) and blurred the information content of the capital ratio for banks' counterparts. The contribution of our paper is to formalize the role of capital requirements over risky investments

[4] It is also important to underline the difference between the risk-shifting literature and the literature on endogenous credit constraints. The latter analyses how asymmetric information introduces external finance premiums and collateral constraints. This literature accounts well for the financial accelerator, either in the boom phase, when the rising price of collateral releases credit constraints (Kiyotaki and Moore, 1997) or in the bust phase, when the collapse in asset prices tightens the credit constraint considerably (Holmstrom and Tirole, 1997). However, these models do not explain why there are equilibria with too much credit and overinvestment in the risky asset.

and show how opacity on the true level of capital pledged by financial intermediaries leads to endogenous uncertainty.

Our paper also shares some common feature with the recent paper by Fahri and Tirole (2009). They study the case where intermediaries do not bear the full cost of their choices because they benefit ex post from a bailout. In their model, the risk-shifting is between intermediaries and the tax payers. We focus instead on the risk-shifting between intermediaries and less informed investors.

The paper proceeds as follows. Section 2 documents the stylized facts about the crisis. Section 3 presents the model. Section 4 solves the model with symmetric information. Section 5 presents the results with asymmetric information. Section 6 is the conclusion. All figures are gathered in section 7.

2 Stylized facts on the pre-subprime crisis

2.1 Debt and risk premia

We dig out two major stylized facts from the literature and from our own observation of the crisis :

1. The risk premium paid by financial intermediaries on their debt has declined.

   US banks benefited from extremely favorable funding conditions during this period not only because of an accomodative Fed's monetary policy, but also because of historical low level of risk premia paid on their debt. A look at the 10 years interest rate spread between commercial paper of US banks and US government bonds[5] (Fig. 1) shows for instance that the premia paid on the risk of banks' default had been non-increasing from 2000 to mid 2007 : the price of credit risk for banks has even declined markedly between 2002 and 2007.

2. The banking sector becomes more leveraged

[5] A similar conclusion can be made from the observation of the spread between Libor and T-Bill rates for shorter maturities (3M, 6M).

Blundell-Wignall and Atkinson (2008) have stressed the huge increase in debt levels (in book value[6]) observed in US banks' balance sheet over the years preceding the crisis. According to Fed's and BEA's figures, the ratio of the debt of the US commercial banking sector over US GDP rose from 59% to 76% between 1999Q4 and 2007Q4. This higher debt level was not associated with tougher capital requirements, and the leverage ratio of the US banking sector (defined as the ratio of debt over equity, at book value) inflated during the same period, from 19 to 44 (see Flow of Funds[7]).

2.2 Evolution of the Regulation

Several reasons explains the favorable norms of capital requirements in this period. For instance, Blundell-Wignall and Atkinson (2008) highlight the difficulty for outsiders to extract extensive information on risk taken by financial intermediaries, the capture of regulator in order to favour easier capital standards, or the procyclicality of the Basel capital regulation framework. Rochet (2008) focuses on the lobbying of the financial industry in the definition of Basel II . He also stresses that the Internal Rating Based (IRB) approach may have delibarately geared regulation toward complexity in the mapping from risky assets to capital requirements. Such complexity can only have favoured interpretations and implementation of capitalisation that would align with the vested interest of the industry. Finally, the accounting rules concerning the consolidation of off-balance sheet entities were incriminated by the Financial Stability Forum for creating "a belief that risk did not lie with arrangers and led market participants to underestimate firms' risk exposures" (April 2008).[8]

[6]Since we discuss the risk of bank's default in the relation between banks' shareholders and lenders, it is more appropriate to consider the banks' liabilities in book value (in opposition with marked-to-market value) in our model framework.

[7]We approximate the leverage ratio as the ratio of total financial liabilities over the difference between total financial assets and liabilities. In particular, our approximation does not take into account non-financial asset and balance-sheet elements relative to non-US area.

[8]This question is actually one of the point in the agenda of the G20 and similar concerns about off-balance sheet vehicules has been brought up by academics (see Acharya and Schnabl 2009), official

We focus on these consensual facts because they are the most relevant to test the model's conclusions and its ability to replicate the build up of financial fragility as we witnessed it during the last decade. We deliberately ignore the dynamics of the crisis itself, including the intertwined collapse of asset prices and fight to quality. Models of these phenomena include Holmstrom and Tirole (1997), Kiyotali and Moore (1997) and He and Krishnamurthy (2008), among others. The main goal of our paper is instead to understand how it was possible for intermediaries to increase their debt level without facing an increasing risk premium.

3 The model

3.1 Markets and assets

Timing

There are 2 dates $t = 1, 2$. Agents make their investment choices at date 1 and get assets returns at date 2.

Financial assets

Three financial assets are available in the economy:

1. A safe asset whose supply X_S is variable, and whose return is r_S. The issuers of this asset get $f(X_S)$ at date 2, where $f()$ is a continuously differentiable function, increasing in X_S though with decreasing return to scale. We assume that the production function is iso-elastic $f(X_S) = X_S^{1-\eta}/(1-\eta)$, but results are robust to the introduction of more general production function.

2. A risky asset in fixed supply X_R, which return is R^*. R^* equals R with probability π and 0 with probability $(1-\pi)$,which is the level of "*economic risk*" in the model. This asset is priced P on the financial market at date 1. The assumption of fixed supply simplifies the model but it can be relaxed provided that the production function of risky assets is not too price sensitive (see Challe and Ragot, 2008).

regulators and central bankers (see for instance speeches of C. Noyer and B. Bernanke in 2008)

We make moreover the following technical assumption

$$\eta > 1 - \pi$$

This assumption is fulfilled for reasonable values of the parameters, as shown below. It insures the uniqueness of the equilibrium.

3. A storage asset S which has a constant return τS. This third asset is available in infinite supply.

Financial assets in this economy can be interpreted in a number of ways:

- The storage asset may for instance represent deposit facilities at the central bank or cash. Indeed, it allows agents to invest without limits at a low and constant rate. In the following, we will use the return on the storage asset as a proxy for the interest rate set by the monetary policy authorities.[9]
- The safe asset accounts for bonds, issued by AAA states or firms. It can be interpreted as a loan to the "real" sector in order to finance investment or production.
- Finally, the risky asset encompasses all types of investments whose expected returns are higher than the return on bonds. It can be either real estate mortgages, junk bonds or stocks

Agents and market segmentation

The economy is composed of three types of agents: households, financial intermediaries and initial sellers.

1. There is a continuum of households, who are risk-neutral and who receive an endowment W^H at the beginning of date 1. The households maximize their date 2 consumption.

[9] Effectively the model is real and the interest rate in the model are real. We assume that monetary policy can affect, possibly only temporarily, the level of real interest rate on the storage asset.

2. There is a continuum of financial intermediaries (that we henceforth also designate as "banks"), who are risk-neutral and who receive an endowment W^F at the beginning of date 1. Intermediaries maximize their consumption over the two periods. In addition, we suppose that intermediaries enjoy a private benefit U from being intermediaries. This benefit will insure that these agents accept to operate as intermediaries rather than consumming all their endowment at period 1.

3. Initial sellers are agents who sell the risky assets to intermediaries at period 1, consume and leave the economy. These agents are only introduced to create a simple supply of the risky asset at the beginning of period 1.

Only financial intermediaries can invest in all the existing assets. Household can only invest in the storage asset or lend to financial intermediaries an amount B at an interest rate r. As Allen and Gale (2000), we introduce this assumption to capture the advanced skills and accumulated rents (asset management abilities, private information and so on) needed to trade corporate bonds and sophisticated financial products. Financial intermediaries never invest in the storage asset, because they have access to the safe asset which yields a higher return.

Financial intermediaries balance sheet and capital requirements

Financial intermediaries' balance sheet is composed of a risky asset PX_R and a safe asset X_S on the asset side, while its liabilities are either equity E or debt B. The amount E stands for the fraction of resources invested by the intermediaries themselves in their business. As we will show in the following section, this amount is directly linked to the intermediaries incentives to take risks. The models can hence describe how bank capital requirements, for instance set by regulation policies, influences these incentives. In particular, the level of capital requirements has a direct bearing on the composition of their portfolio.

Financial regulation imposes that banks invest their own equity for at least Δ per value of unit of risky asset

$$E \geq \Delta PX^R$$

The coefficient Δ is arguably close to the original Cooke ratio of 0.08, although regulation allows for smaller value (Rochet 2008). The balance sheet is

Balance sheet of the financial intermediaries

Assets	Liabilities
X_S	E
PX_R	B

Debt contract

Following Allen and Gale (2000), we assume that households use debt contracts to finance intermediaries and are not able to observe the investment decisions of financial intermediaries. This asymmetric information structure leads to moral hazard because banks do not bear the full costs of the risk incurred by the risky asset. In the bad state of the world, intermediaries file for bankruptcy because the residual value of their portfolio is inferior to their debt. Hence, banks have incentives to take more risks than what would be optimal from the households' standpoint. Households, who anticipate that the possibility to default induces risk shifting, will lend to intermediaries as long as the expected return on this loan remains superior or equal to the return on the storage asset.

Households cannot discriminate between banks because these are identical *ex post*. Hence, households will demand the same interest rate r whatever the size of the loan they grant to the financial intermediary. They supply loanable funds inelastically and the interest rate clears the market.

Information structure.

All agents observe asset prices and the interest rates: the safe interest rate τ, the interest rate r paid by intermediaries and the period-1 price of the risky asset P are known. Intermediaries know the amount of risky asset in the economy X_R and the production function $f(.)$ which produces the safe asset, but households do not know X_R nor the production function $f(.)$. The basic justification for this assumption is that households can easily know the liability side of banks, but it is much more difficult to obtain information about the risk at the asset side. Many observers now recognize that it was difficult even for the rating agencies and financial analysts to assess the real risk born by some restructured assets. This assumption seems thus a natural benchmark for the analysis of risk-shifting in this framework.

We then solve the model with two assumptions concerning the regulatory constraint that intermediaries face. In section 4.2 we assume that households know the exact value Δ. In section 5, we assume that households do not know Δ. As discussed in introduction and in section 2.2, this last assumption is based partly on the unknown extent of regulatory arbitrage.

3.2 Agents

We now describe the problem of each agent.

3.2.1 Households

Households choose the composition of their financial portfolio in order to maximize their final consumption[10], taking market prices, their expectations of aggregate risk and the capital ratio of banks as given:

$$\max_{c,S,B} E\left[c^H\right]$$

where $E[.]$is the expectation operator. The expectaions are made conditional on two different information structures, which are specified in section 4.2 and 5. The households' income constraints are:

$$\begin{aligned} S + B &\leq W^H \text{ (at date 1)} \\ c^H &\leq \rho B + \tau S \text{ (at date 2)} \end{aligned}$$

W^H is the households wealth at the first period. S is the amount invested in the storage asset, and B is the amount lent to intermediairies. For households, the stochastic interest rate ρ that they receive *ex-post* on their loans to financial intermediaries is the only source of uncertainty. If the intermediaries do not default, they get the return defined *ex ante* r. In case of default, households get the residual value of the banks' portfolio $r_S X_S$ so that the *ex post* return per unit of loan is $\frac{r_S X_S}{B}$. Since the loans to financial intermediaries are risky, we define the risk premia paid by intermediaries as the difference between their cost of borrowing r and the riskless rate for household τ.

[10]We could relax this assumption and make households maximise their agregate consumption across periods 1 and 2, at the cost of much more algebra.

Financial intermediaries will always default in the bad state of the world, as in Allen and Gale (2000). Hence, the return on B is

$$\begin{aligned}\rho &= r \text{ with probability } \pi \\ \rho &= \frac{r_S X_s}{B} \text{ with probability } 1-\pi\end{aligned}$$

3.2.2 Financial intermediaries' problem

Financial intermediaries seek to maximize their aggregate consumption at date 1 and 2 with a discount factor β. We assume that

$$\beta < 1/\tau$$

This assumption implies the intermediaries are relatively impatient. They choose their debt level B, and the composition of their portfolio (S, X_S, X_R) taking market prices and the required level of capital as given. Their program is

$$\max_{E,B,X_R,X_S} c_1^F + \beta E\left[c_2^F\right] + U$$

$$\begin{aligned}\text{s.t } c_1^F &\leq W^F - E \\ c_2^F &\leq \max\{R^* X_R + r_S X_S - rB, 0\} \quad (1) \\ PX_R + X_S &= B + E \quad (2) \\ E &\geq \Delta P X_R\end{aligned}$$

In the constraint (1), the max operator indicates the intermediaries' option to default. This option will be considered depending on the value of the stochastic return R^*.

3.2.3 Initial sellers

Initial sellers have no choice and simply consume in period 1 the amount obtained from the sale of the risky asset:

$$c^i = PX_R$$

4 Model resolution

4.1 Pareto Efficient Equilibria

Before solving the model for each of the two information structures above mentioned, we derive the set of Pareto efficient allocations. To do this we maximize the average expected welfare of financial intermediaries for a given expected welfare of households and initial sellers.

This maximization is

$$
\begin{aligned}
& \max_{S,X_s} c_1^F + \beta E c_2^F + U \\
Ec^H &= \bar{c} \\
c^i &= \bar{d} \\
W^f + W^h &= c_1^F + S + X_s + c^i \\
\tau S + f(X_s) + R X_r &= c^H + c_2^F
\end{aligned}
$$

As $\tau < 1/\beta$, the solution to this maximization is simply

$$
\begin{aligned}
S &= 0 \\
f'(X_s) &= \frac{1}{\beta}
\end{aligned}
$$

If there were no market segmentation, each allocation of this set could be reached thanks to appropriate first period lump-sum transfers. In these equilibria, the interest rate on the safe asset would be $1/\beta$ and the price of the risky asset would be equal to the fundamental price:

$$P^* = \beta \pi R \tag{3}$$

4.2 Market equilibrium with known regulation

In this section, we assume that households know Δ.

4.2.1 Financial intermediaries

Intermediaries default in the bad state of the world because their overall return would be negative if they repay their debt (because $B > X_S$).

Their program can be written as

$$\max_{E,B,X_R,X_S} W^F - E + \beta(\pi (RX_R + r_S X_S - rB) + (1-\pi) \times 0) + U$$

$$\begin{aligned} PX_R + X_S &= B + E \\ E &\geq \Delta P X_R \end{aligned}$$

We solve the model under the conjecture that the constraint $E \geq \Delta P X_R$ is binding. This case is of course the relevant one for this model. The binding constraint is equivalent to the following inequality

$$\pi r < 1/\beta \tag{4}$$

This inequality stipulates that expected cost of the debt πr (because debt is repaid only in case of the high return which occurs with probability π) must not be too high. If the expected cost of the debt is too high, intermediaries could want to invest all their wealth to decrease their expected debt burden. Hence, the regulatory constraint would not bind. As r is determined in equilibrium, we show below that the condition (4) is fulfilled for a wide range of parameter values.

The program yields the only asset prices for which the asset market clears.

$$P = \frac{\beta \pi R}{\Delta + \beta \pi r (1-\Delta)} \tag{5}$$

This asset price equilibrium is the main equation of the model. First note that when there is no capital requirement ($\Delta = 0$), the price is simply $P = R/r$ which is the case studied by Allen and Gale (2000).[11] As intermediaries default in case of a bad aggregate shock, their demand for the risky asset is always higher than under the first best equilibrium.

[11] In their model, Allen and Gale show how incomplete debt contracts limit debtors losses in the bad state of the world (losses fall on lenders). In other words, debt contracts act as call options for borrowers. This implies that borrowers only focus on the good state of the world when deciding the composition of their portfolio: the share of the portfolio at risk is higher and the price of risky assets is inflated above its level in a world without segmentation or complete contracts.

Indeed, as $\pi r < 1/\beta$, one finds $P > P^*$. Asset prices are thus too high. Second, when capital requirements increase, the price of the risky asset decreases. Taking r as given, increasing Δ implies a cost in the form of additional foregone consumption in period 1, an effect that dominates the reduction in size of the loan that needs to be repaid with probability π.

Thus, in partial equilibrium, the price of the risky asset can increase for two reasons: either because π increases, which means that the expected return of the risky asset is higher or because Δ decreases (the amount of *ex ante* risk shifting increases). Finally, the demand for the safe asset yields

$$f'(X_S) = r_s \Longrightarrow X_S = [f'(r_s)]^{-1} \tag{6}$$

4.2.2 Households

It has been assumed that the households do not know the amount of risky asset X_R and the risk of this asset, π, but they know the regulatory requirement Δ. One can first show that households can deduce both X_R and π from asset prices and from the value of Δ. Indeed, they can deduce the amount of risk π from the price level (5). Then, they can deduce the amount of aggregate exposure to risk from the amount of regulatory capital and with their knowledge of the coefficient Δ, $E = \Delta P X_R$. Finally, they can infer the amount of safe asset X_S from the budget constraint of the intermediary. To summarize, households can deduce the structure of the asset side of intermediaries and the aggregate risk from the regulatory constraint and from the structure of financial intermediaries' liabilities.

Households anticipate rightly that, in the bad state, the intermediaries' default implies that they get the residual value of the bank's portfolio. With probability π their return per unit invested is r and with probability $(1-\pi)$ it is $r_s X_S/B$. The no-arbitrage condition can be written as

$$\pi r + (1-\pi)\frac{r_s X_S}{B} = \tau \tag{7}$$

4.2.3 Market clearing

First, competition in the financial sector yields $r = r_s$, i.e. the funding cost of the financial intermediaries is equal to the return on the safe asset. This is necessary and sufficient to avoid infinite riskless profit opportunities by the financial intermediaries. In addition, since $\pi \in]0,1[$ and $X^S < B$, the no-arbitage condition (7) implies $r > \tau$. The return on the safe asset X_S for the intermediaries is then strictly higher than the return on the storage asset, and intermediaries never invest in storage.

Equality (7) can be writen as

$$B(r) = \frac{(1-\pi)\, r X_S}{\tau - \pi r} \tag{8}$$

We can substitute E, X_S and P by their equilibrium value given by equations (5) and (6) to find an expression $B\,(r)$.

$$B\,(r) = \frac{(1-\Delta)\, R}{\frac{\Delta}{\beta\pi} + r\,(1-\Delta)} X_R + f'^{-1}\,(r)$$

Using this expression in the equality (7) together with the expression of X_S given by (6), one finds an equation where r is the sole unknown. The solution to this equation gives the equilibrium level of the interest rate.

In this economy, changes in Δ have two effects:

1. a direct effect through the intermediaries incentives to take risk,
2. an indirect one through the evolution of the interest rate r, as households require a higher return when Δ declines.

The increase in the risky asset price will be moderated because r increases. The next section summarizes the effect of a change in Δ.

4.3 Risk-shifting and debt

We perform some comparative statics to analyze how allocation and asset prices change after an increase in *ex ante* risk shifting. The proofs are in appendix A.

Proposition 1 *Both debt level B and the real risk premia $r-\tau$ increase when the capital requirement decrease (i.e. Δ decreases inducing more risk-shifting)*

$$\frac{\partial B}{\partial \Delta} < 0 \text{ and } \frac{\partial (r-\tau)}{\partial \Delta} < 0$$

The debt level of the financial intermediaries B increases when Δ decreases. Reducing Δ has two effects: the demand for B increases and the household supply of B decreases (they require a larger risk premium $r-\tau$). However the overall general equilibrium effect is an increase in the intermediaries debt level since the direct negative effect of Δ on capital requirement dominates other general equilibrium effects through the changing level of B. (see appendix A.2).

However, it should be stressed that when households observe changes in Δ, the effects of risk-shifting on asset prices is somewhat moderated by the response of the risk premium $r-\tau$. Investors realize that the intermediaries take more risks, they require to be compensated. This version of the model is therefore incompatible with the stylised facts of the sub-prime cycle. As showed by figure 1, banks have been able to borrow at lower risk premium during the five years that preceded the crisis, in spite of increasing their leverage and their exposure to US housing assets.

To summarize, if an increase in risk-shifting, which could for instance be due to a larger scale of off balance sheet activities, can account for an increase in the debt level, it cannot explain the path of the risk premia between 2000 and 2007. We therefore assert that risk-shifting per se is not enough to replicate the stylized fact of the *subprime crisis.* Banks and financial intermediaries have effectively benefited from extremely favorable funding conditions before the crisis, a feature that cannot arise in the context of a known regulatory constraint.

5 Fuzzy capital requirements

We now assume that the regulatory constraint is not known by the households. More specifically, households do not know the real value Δ, but receive a signal s about the regulatory constraint which can take two values with equal probability 1/2.

$$s = \begin{cases} +u & \text{prob. } 1/2 \\ -u & \text{prob. } 1/2 \end{cases}$$

We will focus on the case where households overestimate the strength of the regulation $\Delta^H > \Delta$ which is able to reproduce the stylized facts mentionned above.

Households try to infer the relevant values for their portfolio choice from asset prices. We denote the expectations by households of a variable with the upperscript H. Households will thus form the expectations about the aggregate risk level π^H, the quantity of risky asset X_R^H and the quantity of safe asset X_S^H. In this section, we characterize how equilibrium prices and quantities are affected by the households' inference about the financial intermediaries' regulation

First, the price of the risky asset is still given by equation (5) because it results from a no-arbitrage condition for intermediaries, who know the real value π. Households observe the price P the real interest rate r, and have the belief Δ^H and π^H for respectively Δ and π. The following equation must hold:

$$P = \frac{\beta\pi R}{\Delta + r\beta\pi\left(1-\Delta\right)} = \frac{\beta\pi^H R}{\Delta^H + r\beta\pi^H\left(1-\Delta^H\right)} \tag{9}$$

This equation gives the expectation about the level of aggregate risk consistent with anticipated regulatory constraint and from asset prices

$$\pi^H = \frac{1/\beta}{\frac{\Delta}{\Delta^H}\left(\frac{1}{\beta\pi} - r\right) + r} = \frac{\pi}{\frac{\Delta}{\Delta^H}\left(1 - r\beta\pi\right) + r\beta\pi} = \frac{\pi}{\frac{\Delta}{\Delta^H} + \left(1 - \frac{\Delta}{\Delta^H}\right) r\beta\pi} \tag{10}$$

As $r\beta\pi < 1$ in the equilibrium under consideration, the expectations about the level of aggregate risk is higher than the real value π when households overestimate the regulatory constraint.

Second, the no-arbitrage for households gives :

$$\pi^H r + \left(1 - \pi^H\right)\frac{r_S X_S^H}{B} = \tau \tag{11}$$

The households form their expectations of the residual value of their portfolio $\frac{r_S X_S^H}{B}$ in the following way. First, from the observation of the regulatory capital of the banks E

and from their expectations Δ^H, households assume that X_R^H is:

$$E = \Delta\pi X_R = \Delta^H \pi X_R^H \Rightarrow X_R^H = \frac{\Delta}{\Delta^H} X_R$$

From the budget constraint of banks given by equality (2), households form the following expectation of the amount of safe asset $X_s^H = B + E - \pi\frac{\Delta}{\Delta^H}X^R$. Using the equality (9) and the previous equation in the no-arbitrage condition (11), one finds a system of two non-linear equations and the two unknowns: r and π^H. Appendix B provides expressions of these equations as expressions of Δ^H. We show that when households overestimate the regulatory constraint ($\Delta^H > \Delta$) they underrate the level of aggregate risk, $\pi^H > \pi$ and underestimate the amount of risk on the asset side of banks ($X_R^H > X_R$ and $X_S^H < X_S$). The following sections proves that in this case the premium faced by intermediaries can decrease when the amount of debt increases and, more importantly, that the level of the risk free interest rate can amplify mis-pricing.

5.1 A model of the risk-taking channel of monetary policy

This section focuses on the effect of the riskless interest rate on mis-pricing. A number of recent papers have documented that changes in the stance of monetary policy can induce financial intermediaries to adjust their leverage, their credit standards and, more generally their exposure to risks.[12] Borio and Zhu (2009) coined "the risk-taking channel of monetary policy" which they define as *"the impact of changes in policy rates on either risk perceptions or risk-tolerance and hence on the degree of risk in the portfolios, on the pricing of assets, and on the price and non-price terms of the extension of funding"*. However, they also stress that we lack theoritical models of this channel[13].

[12] Adrian and Shin (2008a, b, c and d) show that lower levels of interest rates induce US investment banks to increase their leverage. Ioannou, Ongena and Peydro (2008) and Madaloni, Peydro and Scopel (2009) show evidence that more accomodative monetary policy may imply a deterioration of bank's credit standards. See also Altunbas, Gambacorta and Marques-Ibanez (2009).

Borio and Zhu (2009) discuss the reasons why there may be a "risk taking" channel of monetary policy.

[13] Adrian and Shin (2008) have a model where leverage increases with the liquidity of the repo market. However their model applies more to primary dealer than to commercial banks more generaly. Also, their model is based on the management by banks of their value at risk, where, arguably, the default probabilty

To get some insight on the effects of monetary policy identified as a change in the riskless interest rate, we consider the simplest case where households believe there is no risk of financial intermediaries default ($\Delta^H = 1$). This case is only used here to exhibit the main effects at stake in the model, which is solved in the next section. Households anticipate in this case that $X_S^H = B$ from the budget constraints of banks. As a consequence, they anticipate that the residual value of banks portfolio is high. Using (11) one finds that the equilibrium interest rate is

$$r = \tau$$

As households expect that all the risk is born by banks, the return on the portfolio liquidation is the same as the retun on the safe asset. Hence households charge no premium on banks. The level of aggregate risk deduced from the level of asset prices is

$$\pi^H = \frac{\pi}{\Delta + (1-\Delta)\,\beta\tau\pi}$$

Finally, the price of the risky asset can written simply as

$$P = \frac{\beta\pi R}{\Delta + (1-\Delta)\,\beta\tau\pi}$$

We see immediately that more accommodative monetary policy ($\downarrow \tau$) leads to a rise in P the price of the risky asset. This is interpreted by households as a drop in the aggregate risk (i.e.$\uparrow \pi^H$),. Indeed, households underestimate the effect of a decrease in the riskless rate τ on the price of the risky asset because they underestimate the incentives to take risk. Households do not require premia to compensate for the default risk and the bubbles grows. In what follows, we study the more general case.

5.2 The effects of fuzzy capital requirements

We now assume that households recieve a signal $s = \pm u$ close[14] to 0. The case where $u = 0$ is studied in section 4.2 and yields the equilibrium interest rate r^* charged on

of banks is constant. We focus instead on the response of asset prices to the risk free rate in a context where the default probability of intermediaries is not known by households.

[14] Technically, we focus on small values of u, to take first order approximations. This insures that households form the expectation $\Delta^H = \Delta\,(1+s)$.

intermediaries. We solve for the equilibrium interest rate $r^* (1 + \tilde{r})$. We solve for the deviation to the deterministic case. We also solve for the deviation in the expectations of households of the level of aggregate risk π^H, such that households' expectations are $\pi \left(1 + \tilde{\pi}^H\right)$, where π is the true risk level.

Calculations performed in appendix prove that the expected level of aggregate risk is

$$\tilde{\pi}^H = (1 - \pi \beta r^*) s$$

Two results stem from this expression. First, as $\pi \beta r^* < 1$, when households overestimate the regulatory constraint, $s > 0$, they underestimate the level of aggregate risk $\tilde{\pi}^H < 0$. The intuition for this results has been given in the previous section : Households deduce wrongly from the price of the risky asset a level of aggregate risk which is lower than the real value. Indeed, they underestimate the incentive of intermediaries to take some risk.

The level of the real interest rate appears in this expression. The lower the real interest rate, the higher $1 - \pi \beta r^*$, the larger the underestimation of by households. Indeed, the regulatory constraint interacts with the level of the interest rate r^* in the determination of the price of the risky asset. The lower the interest rate r^* the higher the effect of Δ on the price level, and hence on the level of aggregate risk inferred by households.

To provide a numerical example, we plot the households' beliefs about aggregate risk π^H as a function of the signal s . We take the following numerical values $\beta = 0.96$, $R = 1$. We also set $\pi = 0.8$ and $\Delta = 0.08$.

Figure 2 shows that households underestimate the risk in the economy $\pi^H > \pi$ when they overestimate the strength of regulatory constraints $s > 0$. Besides, they always infer the right probability $\pi^H = \pi$ when the signal they receive is not noisy $(s = 0)$, whatever τ. Finally, a lower monetary policy (a low τ) increases the size of the error, for the reason given in the previous section.

The linearization yields an equilibrium interest rate

$$\tilde{r} = -G(\tau) s$$

Unfortunately, the expression $G(\tau)$ cannot be easily interpreted because of the various effect at stake. First, the expectations about the aggregate risk $\tilde{\pi}$ have a direct negative effect on risk premia and hence on the interest rate $\tilde{r}$. Second, the change in the riskless interest rate τ affects the equilibirum price and the budget constraint of the banks. As a consequence, the residual value of the banks' portfolio in case of a bad aggregate shock is affected.

To exhibit the result of these effects, we perform a following simple calibration exercise. We take the following numerical values $\beta = 0.96$, $R = 1$. We also set $\pi = 0.8$ and $\Delta = 0.08$.

Figure 3 shows that the equilibrium risk premium $r^* - \tau$ is an increasing function of the safe return τ. As a consequence, the risk premium falls when the safe interest rate decreases because households anticipate that the level of aggregate risk is diminishing. The effect of the safe interest rate on the risk premium is represented in Figure 4, where the effect of the signal (the coefficient $G(\tau)$) is plotted. When the safe interest rate decreases, the effect of a same signal is amplified on the interest rate paid by intermediaries. In other words, the same overestimation of the regulatory constraint yields a higher reduction in the risk premium when the riskless interest rate decreases.

What are the implications of the risk taking channel of monetary policy for policy makers?

Our model suggests that it may be desirable to change the level of interest rates to fulfil a financial stability objective in the circumstances when capital requirements appear deficient and cannot be adjusted directly. Such a situation can arise for instance when capital requirements cannot be adjusted unilateraly in one country because they are typically negociated between countries over several years. However, even in such circumstances, the monetary authorities should act only if reasonably convinced either that the level of capital requirements is not well perceived by investors or that a class of investors underestimate the true level of risk.

6 Concluding remarks : Can the model explain the build up of financial fragility?

In this paper, we showed first that the combination of risk-shifting and fuzzy capital requirement can explain one of the most puzzling stylized fact of the sub-prime crisis i.e.

that banks could ever increase their exposure to risk without having to pay higher risk premia on their debt.

In a situation of uncertainty with respect to regulatory constraints, the increase in the observed asset prices can be interpreted as a lower aggregate risk in the economy while, effectively, asset prices were driven by greater risk-taking by financial intermediaries. We also showed that this model give rise to a risk-taking channel of monetary policy: the influence of the level of interest rate on risk perception by some agents and exposure to risks by others.

Our result extends the popular intuition explaining how the signal extracted from market price is polluted by noise coming from excessive risk-taking behavior when the mapping of capital requirement is not observable by agents. In our model, market forces are not able to lead by themselves the economy to the optimum allocation of capital because incentives are not correctly understood.

We see two obvious extensions to our model. First, it is possible to endogenize the expectations of households in a dynamic setting where households learn about the relevant parameters. Although, the results of our models would hold if the prior of the households are far enough from the true parameters, the resulting dynamics may lead to interesting patterns. Second, it would be interesting to study the political economy associated with the assessment of risk within the economy. Sellers of the assets have incentives to underestimate the level of economic risk or to generate complexity to increase the cost of signal extraction.

This should be anticipated by households who then would look for other sources of information. We understand the current discussion about rating agencies as a part of the political economy debate on the management of risk expectations in economies where intermadiaries play an important role.

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7 Figures

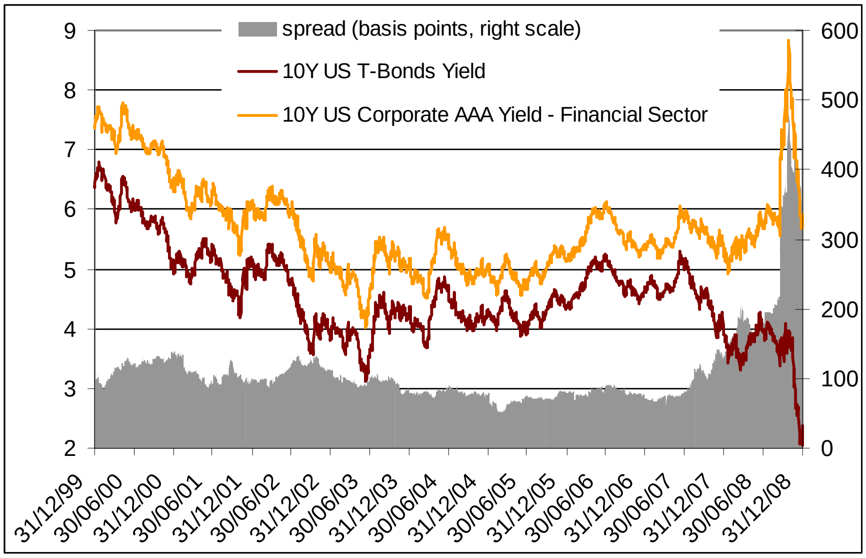

Fig 1: Spread between 10Y US T-Bonds and 10Y Bonds of US AAA Financial Companies

Source: Bloomberg

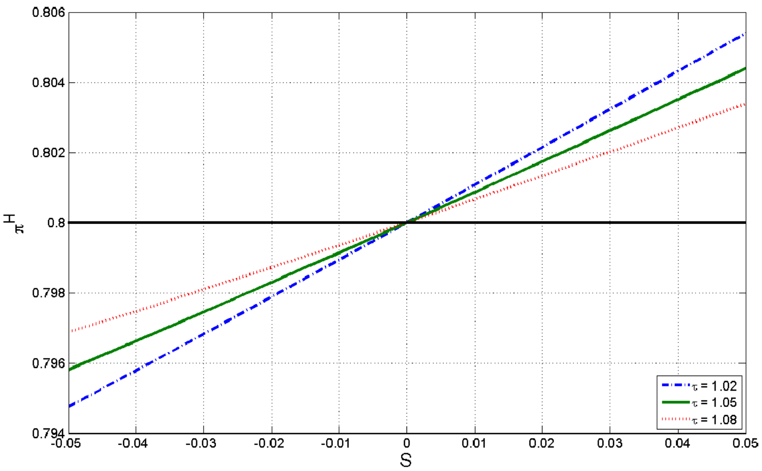

Fig. 2:Households′ expectation of aggregate risk π^H as a function of s

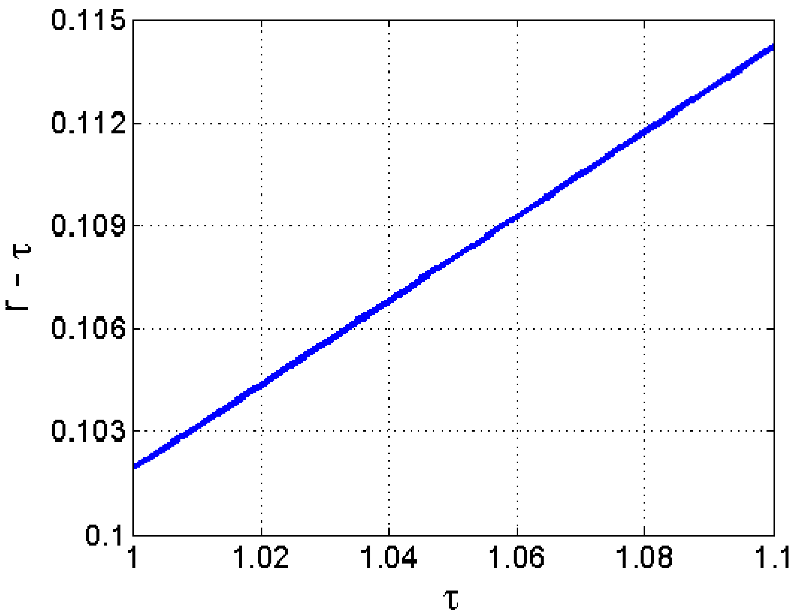

Fig. 3: Risk premium $r^* - \tau$

as a function of τ

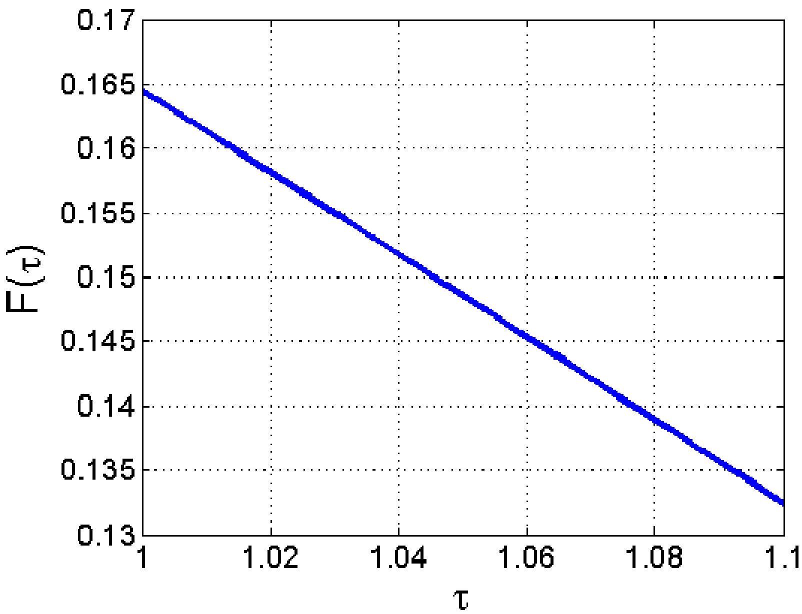

Fig. 4: Coefficient $G(\tau)$

as a function of τ

8 Appendix

A Equilibrium with Symmetric Information

A.1 Interest Rate: $\frac{\partial(r-\tau)}{\partial\Delta} < 0$

Recall that:

$$B = (1-\Delta)PX_R + X_S = \frac{(1-\Delta)\,R}{\frac{\Delta}{\beta\pi} + r\,(1-\Delta)}X_R + f'^{-1}\,(r) \tag{12}$$

and

$$\tau = \pi r + \frac{1-\pi}{B} r f'^{-1}\,(r) \tag{13}$$

Let us define: $\Theta = \frac{\Delta}{1-\Delta}$. Then from (12)

$$B = \frac{R}{\frac{1}{\beta\pi}\Theta + r}X_R + f'^{-1}\,(r)$$

and (13)

$$\begin{aligned} \tau &= r\left(\pi + (1-\pi)\,\frac{f'^{-1}\,(r)}{\frac{R}{\frac{1}{\beta\pi}\Theta + r}X_R + f'^{-1}\,(r)}\right) \\ \Rightarrow &\quad \Theta = \beta\pi\left(\frac{R\,(\tau - \pi r)}{1-\frac{\tau}{r}} r^{\frac{1}{\eta}-1} X_R - r\right) \end{aligned}$$

The last equality defines a function $\Theta\,(r)$, which gives the value of Θ (and hence Δ) necessary to obtain an equilibrium interest rate r. We prove that $\Theta\,(r)$ is decreasing, and hence that the function $r\,(\Theta)$ is decreasing. Diffenciating $\Theta\,(r)$, a sufficient condition to obtain $\Theta'\,(r) < 0$ is

$$\left(\frac{1}{\eta} - 1\right)\left(\frac{\tau}{r} - \pi\right) < \pi + \frac{\tau}{r}\frac{\tau/r - \pi}{1-\tau/r}$$

Define $x \equiv \tau/r$. Along the equilibrium under consideration $\pi < x < 1$. After some algebra, one finds that a sufficient condition is

$$\eta > 1 - \pi$$

The condition is satisfied for instance for $\eta > 1/2$ and $\pi > 1/2$. In this case, r is decreasing with Δ. CQFD

A.2 Aggregate Debt: $\frac{\partial B}{\partial \Delta} < 0$

From equality (7), one finds

$$B = \frac{1-\pi}{\tau - \pi r} r^{1-\frac{1}{\eta}}$$

After some algebra, a sufficient condition for B to increase with r is

$$\eta > 1 - \pi$$

If the previous condition is fufilled one finds $\frac{\partial B}{\partial r} > 0$ and $\frac{\partial (r-\tau)}{\partial \Delta} < 0$. As a consequence, $\frac{\partial B}{\partial \Delta} < 0$.

B Equilibrium with uncertain regulation

Using the budget constraint of intermediary, the residual value of the investors portfolio in case of default can be written as

$$\left(1 + \Delta\left(1 - \frac{1}{\Delta^h}\right) \frac{1}{1 - \Delta + \left(\Delta \frac{1}{\beta\pi} + (1-\Delta) r\right) \frac{f'^{-1}(r)}{R}}\right)$$

As a consequence, the no-arbitrage condition for households is

$$\pi^h r + \left(1 - \pi^h\right) r \left(1 + \Delta\left(1 - \frac{1}{\Delta^h}\right) \frac{1}{1 - \Delta + \left(\Delta \frac{1}{\beta\pi} + (1-\Delta) r\right) \frac{f'^{-1}(r)}{R}}\right) = \tau \qquad (14)$$

The expectations about the aggregate risk is given by equality (**??**):

$$\pi^h = \frac{1}{\frac{\Delta}{\Delta^h}\left(\frac{1}{\pi} - \beta r\right) + \beta r}$$

Substituting $\Delta^H = \Delta(1+u)$, and linearizing with respect to u, the previous equation yeilds

$$\tilde{\pi}^H = \left(1 - \pi\beta r^{rs}\right) u$$

Linearizing equality (14) yields

$$\tilde{r} = -\frac{\frac{1}{1-\Delta} + \frac{\pi}{1-\pi}\left(1 - \pi\beta r^{rs}\right)}{\frac{1}{1-\pi} + D\frac{1}{R}\left(r^{rs}\right)^{-\frac{1}{\eta}}} u$$

with

$$D = \frac{\Delta \frac{1}{\beta\pi} + (1-\Delta)\, r^{rs}}{(1-\pi)(1-\Delta)} - \frac{\left(\frac{1}{\eta} - 1\right)(1-\Delta)\, r^{rs} + \frac{1}{\eta}\frac{1}{\beta\pi}\Delta}{1 - \Delta + \left(\Delta \frac{1}{\beta\pi} + (1-\Delta)\, r^{rs}\right)\frac{1}{R}\left(r^{rs}\right)^{-\frac{1}{\eta}}}$$