Minimal Consumption Threshold, Persistent Inequality and Development

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Abstract

This paper develops a theory of the evolution of income that is consistent with one of the most striking regularity : the reallocation of the labor force toward the sector of services. In our model, early stages of industrialization are characterized by a rise in inequality because the rich leave a bequest while the poor devote their wealth to consumption of agricultural and manufactured goods. In the model, capital accumulation by the rich class increases real wealth of poor individuals. This allows them to break away from their constraint of minimal consumption and to start consuming luxury goods as services. This sector being labor intensive, capital accumulation implies that labor force moves toward the sector of services.

We also exhibit conditions under which inequality can persist. For instance, if rich class accumulation is not sufficient, stable steady state is inegalitarian. The effect of redistribution is then studied.

Keywords: Income Distributions, Growth, Engel's law, Structural Change

JEL classification O11, O15, O41, D9

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1 Introduction :

This paper aims to provide an adequate theory of evolution of income distribution that links differences in saving behaviors among social classes with one of the most striking regularities of the growth process : the massive reallocation of labor from agriculture into services.

This reallocation process often called "structural change", has been documented by authors such as Clark (1940), Kuznets (1957), and Chenery (1960). A few figures help to put this phenomenon into perspective. In 1870, the U.S. share of employment in agriculture was 40%. One hundred years later, agriculture accounted for only 4% of employment. Services, which accounted for 20% of employment in 1870, absorbed 40% of the labor force by 1970[1]. For the U.K., Dean and Cole (1969, p.137 and Table 30) shows that agricultural, forestry and fishing employment shrinks from 60-80% of total labor in the late seventeenth century, to 30-40% in 1800, 20-25% in 1850, 8-10% in 1900, and 5% by 1950. Meanwhile, employment in services rises in the same way as in the US. Moreover, the sectorial movements documented by the above analysis are not peculiar to the U.S. or the U.K. economy. Kongsamut *et al* (1999) studies both a long-run data set comprising 22 countries and a cross-section data set of 123 non-socialist countries for the period 1970-1989. Both data sets confirm the development regularities: growth in *per capita* income tends to be accompanied by a rise in services and a decline in the agricultural sector, both in terms of labor employment and relative weight in GDP. In addition, average saving rates tend to be historically higher after the structural change than before. An older literature emphasizes that a society may be able to change its saving behavior over time. For example, Lewis (1954, p.155) writes, "The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4-5% of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15% of national income or more".

Our analysis provides a new theory of evolution of income that is consistent with the above striking regularities. It is based upon *two fundamental elements* that are well supported by

[1] Kongsamut, Rebelo and Xie (2001) sum up the main elements of the structural change that has taken place in the US and in other growing economies during the last 100 years. They provide a related theoretical reproduction of this phenomenon.

empirical evidences. *First*, the marginal propensity to save and to bequeath increases with wealth (see for example Dynan, Skinner, and Zeldes (2004) and Tomes (1981)). This fact is encompassed in our model through the assumption that individuals' preferences are consistent with the Engel's law. In other words, individuals choose to devote an increasing part of their wealth to bequests. In particular, we consider that individuals must consume a minimum level of primary goods (mainly agricultural and manufactured goods)[2] before consuming luxury goods (mainly services). This is based upon ideas firstly developed by Kaldor (1955) and Pasinetti (1962). Indeed, Kaldor (1955) develops a model where saving rates are different according to the type of income (wages and profits). In contrast, the literature on individual differences in savings behavior firstly introduced by Pasinetti (1962) and further elaborated by Meade (1964) and Stiglitz (1969) models differences in saving behavior between workers and capitalists, rather than between wage-earners and profit-earners. Surprisingly, Stiglitz (1969) shows that in the standard Solow model, convergence toward an equal distribution is ensured even if there exists differences in saving behavior. Indeed our model endogeneises these assumptions and saving rates will evolve through capital accumulation (initially, capitalists save more than workers because they are richer and can afford their minimal consumption). Our model will thus contrast with several results of this literature, in particular, as will become clear, convergence toward an equal distribution will not hold. *The second important feature* is the following: throughout the process of development, all prices exhibit a decreasing trend and the price of primary goods decreases faster than the price of other goods and more particularly than luxury goods. This last evidence relies on several old works and the majority of recent empirical studies. The Bosserelle, Jubenot, Rasselet (1996)'s study does an inventory of these different thesis and gives robust stylized facts for France and the United Kingdom (see also Gayer, Rostow and Schwartz (1953), Lewis (1952) and Lewis (1978)). This point will be encompassed by our model and will be revealed as a key feature for the understanding of evolution of inequality.

Therefore, our model relies on the interaction between a greater marginal propensity to save and to bequeath for rich than for poor individuals, and an increasing (respectively decreasing)

[2] Intuitively primary goods are such that an individual will not be interested in any other goods if he has not reached a minimal threshold of consumption for these goods. One can think to lexical order between primary goods and any other goods. Food, housing or basic clothing can be seen as primary goods.

trend of luxury goods / primary goods (respectively primary goods / luxury goods) relative price. In early stages of development only rich individuals' inheritance increases along the time path. Nevertheless, the rich individuals' behavior, *i.e.* an increase in intergenerational transfers, implies an aggregate savings increase, and consequently a rise in aggregate capital stock. Therefore, this tendency has two implications. On the one hand, it increases marginal productivity of labor and so increases workers' wages. On the other hand, since the sector of primary goods is capital intensive, capital accumulation raises output in this sector and so, exerts a downward pressure on primary goods price (relatively to the luxury goods price)[3]. Both effects lead to an augmentation of real wealth of poor individuals. This real wealth increase will allow the poor to break away from the constraint of primary goods consumption. Consequently, they will be able to adopt other consumption behaviors. Especially, they will start to bequeath and, in the manner of rich individuals, enter in an intergenerational wealth accumulation process (for the sake of simplicity, this will be called the second stage of development). Since capital accumulation (together with capital intensiveness of primary goods sector) leads to a transfer of the labor force towards luxury goods sector (considered as the sector of services), it is clear that our model reproduces the reallocation process of the "structural change" described above. We can also infer that the second stage of development is characterized by an important rise in aggregate savings.

Interestingly, our model exhibits under certain conditions, persistent inequality even when there exists no credit market imperfection. Our persistent inequality result, within the framework of a perfectly competitive economy, is due to the fact that the real wealth increase can be too small. Thus, in the long-run, poor individuals devote their entire wealth to the consumption of primary goods. Consequently, the long-run equilibrium is an inegalitarian steady state. The economy will reach its long-run egalitarian equilibrium under the condition that primary goods price fall sufficiently.

Note that in our model, an increase in inequality implies an increase in every sector output

[3]Through learning by doing, in the perspective of Romer (1986), capital accumulation could lead to another implication: an increase in technological progress. A slightly different version of the model (subject to a decrease of marginal productivity on aggregate capital stock) could encompass this feature, but this will not change our results.

and an earlier inequality reversal. This is due to the fact that such an "anti-redistribution" channels resources in the hand of people having the higher propensity to save and the fastest speed of accumulation. More inequality also leads to a lower probability of remaining mired in an under-development trap. As in several recent papers (e.g. Galor and Tsiddon (1997)), the study therefore suggests that an under-developped economy, which values equality as well as prosperity, may confront a trade-off between equality and stagnation in the long-run and inequality in the short-run followed by equality and prosperity in the long-run. Nonetheless, we will see that this trade-off is not a necessity. In particular, the government could use government savings to speed accumulation and hence structural change. Moreover, such a policy would also allow an economy to get off an under-development trap.

The remainder of this paper is organized as follows. Section 2 provides the basic structure of the model. Sections 3-1 to 3-4 supplies conditions under which our model reproduces evidences on development such as the structural change and the increase in aggregate savings. Section 3-5 is divided into two parts. First we characterize conditions under which persistent inequality occurs. Then we analyze the impact of redistribution within this framework where there exists a minimal consumption threshold.

2 The Basic Structure of the Model

2.1 Production of Final Outputs

The production of the two heterogeneous goods respectively, denoted by indices 1 and 2, occurs within a given period according to neoclassical constant-returns-to-scale production technologies. Good 1 shall henceforth be referred to as a primary goods (as specified earlier, these are agricultural and manufactured goods) whereas good 2 will be referred to as luxury goods (or services). Such a terminology shall be justified later on.

The primary goods output $Y_{1,t}$ is produced at time t according to:

$$Y_{1,t} = AK_t^{\alpha} L_{1,t}^{1-\alpha}, \alpha \in (0,1). \tag{1}$$

where K_t and $L_{1,t}$ are the quantities of physical capital and labor employed in production of the primary good at a given period t. A is a scale parameter.

The production of luxury goods is made exclusively from the labor input. In order to facilitate the model resolution, we adopt a formulation of production technology in this sector, with constant-returns-to-scale, and a unique factor of production (labor). Nevertheless the key feature relies on the fact that production of primary goods is more capital intensive than production of luxury goods. This point is well supported by empirical analysis. Baldwin (1956) pointed out that, since the first half of the nineteenth century, luxury goods were mostly handmade. Whereas standardized mass-products such as primary goods use mass production technique which is strongly capital intensive. The amount of luxury output produced at time t, $Y_{2,t}$ states as :

$$Y_{2,t} = DL_{2,t}. \tag{2}$$

where $L_{2,t}$ is the quantity of labor employed in the production of this good at time t, and $D > 0$ is a technology parameter in this sector. Labor force is constant and normalized to 1. Producers operate in a perfectly competitive environment.

In the sequel, good 1 is retained as the *numéraire* at every period. Thus, P_t denotes the (relative) price of goods 2 (to goods 1). Moreover, it is assumed that capital is produced[4] in sector 1. Given the wages rate w_t in terms of good 1, in period t producers of sector 2 choose the level of labor so as to maximize profits :

$$\pi_{2,t} = P_t Y_{2,t} - w_t L_{2,t}. \tag{3}$$

That is :

$$w_t = DP_t. \tag{4}$$

Sector 2 is thus characterized by a constant real wage which is equal to the marginal productivity of labor. Denote R_t for the factor gross return of the capital stock and r_t for the rate of return (i.e., we have $R_t = (1 + r_t)$). We assume that the rate of depreciation of the capital δ is equal to 1 (capital fully depreciates on use at each period of time). The instantaneous profit of firm 1 in period t can be written :

$$\pi_{1,t} = Y_{1,t} - \delta K_t - r_t K_t - w_t L_{1,t} = Y_{1,t} - (1 + r_t)K_t - w_t L_{1,t}$$

[4] Assuming that capital is produced by sector 2 and therefore by a technology using only labor would be unrealistic.

or,

$$\pi_{1,t} = Y_{1,t} - R_t K_t - w_t L_{1,t}. \tag{5}$$

Profit maximization leads to the following optimality conditions:

$$R_t = \alpha A[L_{1,t}/K_t]^{1-\alpha}. \tag{6}$$

$$w_t = (1-\alpha)A[K_t/L_{1,t}]^{\alpha}. \tag{7}$$

Under constant-returns-to-scale, those two conditions lead to the factor price frontier, which can be expressed, from equations (6) and (7) as:

$$R_t = \alpha A[\frac{A(1-\alpha)}{w_t}]^{(1-\alpha)/\alpha}. \tag{8}$$

Finally and from (6) and (7), the physical capital-labor ratio demand of firm 1 is given by :

$$\frac{K_t}{L_{1,t}} = \frac{\alpha}{(1-\alpha)}\frac{w_t}{R_t}. \tag{9}$$

2.2 Individuals

In every period, a generation made of a continuum of individuals of measure 1 is born. Each individual has a single parent and a single child. Individuals, within as well as across generations, are identical in their preferences and innate abilities. An *ex-post* heterogeneity between individuals is introduced by differences in their initial wealth and is maintained throughout time because of differences in their optimal choices.

We assume that the economy initially consists of two groups of individuals. Those two groups differ only from the initial bequest they have inherited from their parents - Rich denoted by R are a fraction μ of the population and Poor denoted by P are a fraction $1-\mu$. Since individuals are *ex-ante* homogenous within a group, the uniqueness of the solution to their optimization problem ensures that their offsprings are homogenous as well.

2.2.1 Wealth and Preferences :

Individuals live for two periods. An individual i born in t (a member i of generation t) saves the bequest that was left by his parent, namely: b_t^i, it is entirely made in good 1. In the second period of their life-span (at $t+1$ - his adulthood), this individual supplies one unit of labor at the competitive market wage, w_{t+1}. In addition, the individual receives the return on the parental bequest $R_{t+1}b_t^i$. The individual then allocates this wealth between consumptions (of good 1 and 2), and bequests to his offspring. Note that our model is different from the standard Diamond-Samuelson model where agents work in period one and retire in period two. In particular, we assume that children consumption is included in the consumption of parents. We adopt a form of altruistic bequest motive that is known as "joy of giving" (*i.e.* the amount of bequest appears as an argument of the utility function). This is the common form in recent literature on income distribution and growth. It is supported empirically by Tomes (1981).

We denote the individual's second period wealth by I_{t+1}^i. The individual second period budget constraint (in period $t+1$) states as:

$$C_{1,t+1}^i + P_{t+1}C_{2,t+1}^i + b_{t+1}^i \leq w_{t+1} + R_{t+1}b_t^i \equiv I_{t+1}^i. \tag{10}$$

The preferences of a member i of generation t are represented by a utility function that captures the spirit of Engel's law [5]:

$$U(C_{1,t+1}^i, C_{2,t+1}^i, b_{t+1}^i) = \left\{ \begin{array}{c} C_{1,t+1}^i \text{if } C_{1,t+1}^i < \bar{C}_1 \\ (C_{2,t+1}^i)^{1-\beta}(b_{t+1}^i)^{\beta} + \bar{C}_1 \text{ if } C_{1,t+1}^i \geq \bar{C}_1 \end{array} \right\}, \tag{11}$$

for $i = P, R$.

Below a certain threshold of consumption, *i.e.* $\bar{C}_1$, an individual only gets satisfaction from the consumption of primary goods. The individual will rationally allocate his entire wealth to the consumption of the primary goods, forsaking consumption of any other good, and more particularly of bequest. Such a specification for preferences is consistent with Engel's law according to which the marginal propensity to consume primary goods is a decreasing function of wealth.

Once this threshold of minimal consumption $\bar{C}_1$ is reached, the individual stops consuming new primary goods. The only way for him to reach a greater satisfaction will consist in consuming

[5] This utility function is close to the one used in Murphy Shleifer, Vishny (1989). The resulting behavior in terms of bequest is empirically confirmed by Tomes (1981). This is a streamlined characterization of Engel's law. Although Engel's law is usually formulated in terms of expenditure shares, note, for example, that according to Houthakker's (1987) description of it, "there is also evidence that the income elasticity of food, like the budget share, is inversely related to income; the elasticity may be as high as 0.8 or 0.9 at very low income levels, and close to zero for high income.

luxury goods and transferring bequest to his offspring. He will allocate his remaining wealth between bequest and luxury goods.

2.2.2 Optimal Consumptions and Transfers

In period $t+1$, a member i of generation t chooses the level of each consumption and of transfers to his offspring. The agent maximizes the utility function subject to his intertemporal budget constraint, that is :

$$max_{\{C^i_{1,t+1},C^i_{2,t+1},b^i_{t+1}\}} U(C^i_{1,t+1}, C^i_{2,t+1}, b^i_{t+1})$$
$$s.t.(10).$$

Two sets of optimal conditions appear as being admissible :

First, the individual happens to be too poor to consume more than $\bar{C}_1$: he will devote his entire wealth to primary goods and will not consume any luxury goods. Moreover, he will not leave any bequest to his offsprings. This case occurs when $I^i_{t+1} \leq \bar{C}_1$, and implies that: $C^i_{1,t+1} = I^i_{t+1}$, and $C^i_{2,t+1} = b^i_{t+1} = 0$. The other case occurs when an individual is sufficiently rich so as to finance a threshold level of consumption of primary goods, then his consumption of primary goods will be $C^i_{1,t+1} = \bar{C}_1$. He will then be in position to bequeath and to consume luxury goods, this leads to $C^i_{2,t+1} = (1-\beta)\frac{[w_{t+1}+b^i_t R_{t+1}-\bar{C}_1]}{P_{t+1}}$, and $b^i_{t+1} = \beta[w_{t+1} + b^i_t R_{t+1} - \bar{C}_1]$. Such a case appears when $I^i_{t+1} > \bar{C}_1$.

To sum up, consumption choices can be synthesized in the following form:

$$C^i_{1,t+1} = \min\{I^i_{t+1}; \bar{C}_1\}, \tag{12}$$

$$C^i_{2,t+1} = \max\{(1-\beta)\frac{[w_{t+1} + b^i_t R_{t+1} - \bar{C}_1]}{P_{t+1}}; 0\}, \tag{13}$$

$$b^i_{t+1} = \max\{\beta[w_{t+1} + b^i_t R_{t+1} - \bar{C}_1]; 0\} \tag{14}$$

Therefore, optimal conditions to individuals' maximization of utility can be characterized by corner solutions. This property implies that the equation of bequest accumulation can be different depending upon the wealth levels of the individuals. In our case, accumulation function of the poor is degenerate in the sense that they leave a nil bequest.

2.3 General Characterization of Dynamics

We have assumed that our economy is characterized by two groups of homogenous individuals: rich and poor. Both groups can be distinguished by their initial level of bequest and their size is constant over time. We now define two groups of individuals being distinguished by the fact that they are (or are not) in a position to bequeath to their offspring; at any date, size of any of the two groups depends upon their respective wealth. We denote by λ_t the fraction of individuals who are able to bequeath *at date* t, and by $1 - \lambda_t$ the fraction of individuals unable to leave a bequest *at the same date* t.

We first treat the dynamics in a general way, from which we will infer the specific dynamics of the two main regimes. As will be revealed in our argument, our two regimes will be associated to distinct values of λ_t. The first regime will be characterized by the impossibility for the poor to bequeath, the value of λ_t being given by the number of rich individuals, *i.e.* $\lambda_t = \mu < 1$. By contrast, the second regime corresponds to a value of λ_t equal to 1, poor and rich individuals being both able to bequeath. Thus we will be interested in dynamics for constant value of λ_t.

At the capital market equilibrium, the aggregate stock of capital is equal to aggregate savings. The following equation describes the equilibrium of the capital market, (the right-hand side is the savings that is equal to the aggregate value of bequest while the left-hand side is equal to the investment):

$$K_{t+1} = \int_0^1 b_t^i di \tag{15}$$

The equilibrium of the luxury goods market describes the equality between luxury goods production and aggregate luxury goods demand, that is :

$$DL_{2,t+1} = \int_0^1 C_{2,t+1}^i di \tag{16}$$

Finally, the labor market equilibrium states as:

$$L_{1,t+1} + L_{2,t+1} = 1 \tag{17}$$

Thus, we can define our intertemporal equilibrium as :

Definition 1 *A nonnegative sequence* $\{K_{t+1},\ L_{1,t+1},\ L_{2,t+1},\ w_{t+1},\ R_{t+1},\ C_{2,t+1}^R,\ b_{t+1}^R,\ C_{1,t+1}^R,$

$C^P_{2,t+1}$, b^P_{t+1}, $C^P_{1,t+1}$, $P_{t+1}\}_{t\geq -1}$ *constitutes an intertemporal equilibrium for our model if it is consistent with*

(i) b^i_{-1} *given for* $i = P, R$

(ii) household maximizations given by (12), (13), and (14) for every individual $i = P, R$

(iii) profit maximizations of both firms synthesized by (4), (8) and (9)

(iv) equations describing market equilibria are all to be satisfied[6]*: (15), (16) and (17).*

The following equation defines a static and increasing monotonic relation between K_t and P_t (the reader will find details in appendix 1).

$$\{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{\alpha}}\}K_t = [1-\lambda_t(1-\beta)](DP_t)^{1/\alpha} + \lambda_t(1-\beta)\bar{C}_1(DP_t)^{\frac{1-a}{\alpha}}. \tag{18}$$

This increasing relationship between capital stock and price of goods 2 (in terms of goods 1) can be easily understood. When capital stock increases, it exerts an upward pressure on the marginal productivity of labor in sector 1. Therefore, sector 1 wages increase and this creates incentives for workers to offer their labor force to sector 1. Production in sector 1 then increases while production in sector 2 decreases. To restore equilibrium, the relative price has to increase.

We now present the equilibrium dynamics (see Appendix 1 for calculus):

$$\begin{aligned} &\frac{[1-\lambda_{t+1}(1-\beta)](DP_{t+1})^{1/\alpha} + \lambda_{t+1}(1-\beta)\bar{C}_1(DP_{t+1})^{\frac{1-\alpha}{\alpha}}}{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{(\alpha}}} \\ = \;& \beta\lambda_t DP_t - \beta\lambda_t\bar{C}_1 + \frac{\beta\alpha}{1-\alpha\beta}\{[1-\lambda_t(1-\beta)]DP_t + \lambda_t(1-\beta)\bar{C}_1\}. \end{aligned} \tag{19}$$

This equation implicitly relates P_{t+1} and P_t, to save on notation

$$LHS(P_{t+1}, A, \lambda_{t+1}) = RHS(P_t, \lambda_t).$$

In this section we study the properties of dynamics assuming λ_t to be constant. This will be the case in our two regimes. The graph of a possible dynamics is depicted in figure 1. At period 0, the first adults earn a bequest b^i_{-1} giving K_0: but as we have shown above, we have a static and increasing monotonic relation between K_t and P_t , this gives us the initial condition[7] P_0.

[6] We omit the equilibrium equation of primary goods since it is ensured by Walras' law.

[7] In other words, P_t is a backward variable.

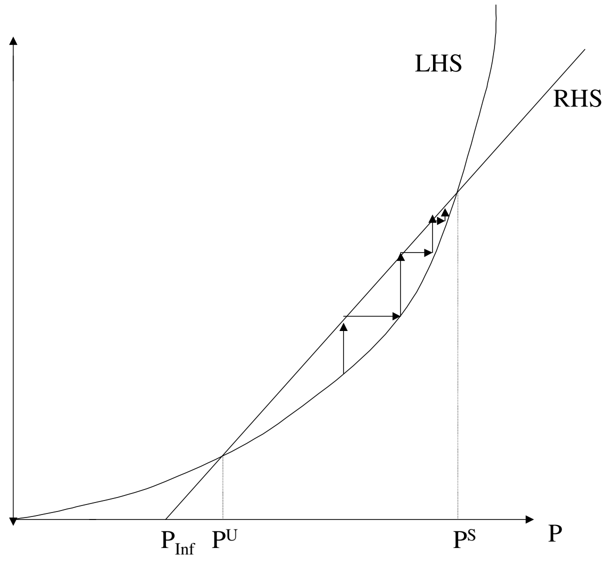

Figure 1

We now provide some properties of RHS and LHS :

Lemma 1 *(i)* $\frac{\partial LHS}{\partial \lambda} > 0 \Longleftrightarrow P < \frac{\bar{C}_1}{D}$*;* $\frac{\partial LHS}{\partial P} > 0$ *for all* $P > 0$*; if* $\alpha < \frac{1}{2}$ *then* $\frac{\partial^2 LHS}{\partial^2 P} > 0$; *if* $\alpha > \frac{1}{2}$ *then* $\frac{\partial^2 LHS}{\partial^2 P} > 0 \Leftrightarrow P > \frac{\lambda(1-\beta)\bar{C}_1(2\alpha-1)}{[1-\lambda(1-\beta)]D}$*; finally,* $LHS(0) = 0$*.*

(ii) $\frac{\partial RHS}{\partial \lambda} < 0 \Longleftrightarrow P < \frac{\bar{C}_1}{D}$*;* $\frac{\partial RHS}{\partial P} > 0$ *for all* $P > 0$*;* $\frac{\partial^2 RHS}{\partial^2 P} = 0$*; finally* $RHS(P_{inf}) = 0$ *where* $P_{Inf} = \frac{\beta\lambda\bar{C}_1[1-\frac{(1-\beta)\alpha}{1-\alpha\beta}]}{D\{\beta\lambda+\frac{\beta\alpha[1-\lambda(1-\beta)]}{1-\alpha\beta}\}} > 0$.

Proof. The omitted proof comes from some simple algebra. ■

Let us state the main properties of our dynamics :

Lemma 2 *(i) To any* P_t *in* $[P_{Inf}, +\infty)$ *is associated a unique* P_{t+1}*, (ii) the dynamics are monotonous.*

Proof. (i) Given $P_t \in [P_{Inf}, +\infty)$, the right hand side of equation (19) corresponds to a positive constant, but there exits a unique P_{t+1} such as the left hand side is equal to this positive constant. Therefore to each P_t, is associated a unique P_{t+1}. (ii) From equation (19) we see that $LHS(P, \lambda, A)$ and $RHS(P)$ are increasing functions in P_t. Thus, $G = LHS^{-1} \circ RHS(P_t)$ is an increasing function. ■

P_{Inf} can be seen as the price under which rich individuals are not able to leave any bequest since their real wealth is too low[8].

We shall then study the existence, uniqueness and stability of the steady states defined as fixed points of the function $G(.)$. Three configurations can appear: either no steady state exists, either there exist two steady states among which one is stable and one is unstable, or there can exist a unique steady state which is stable on its right and unstable on its left. Without loss of generality, this last case will not be explicitly considered.

Steady state existence is of course a crucial issue in our model. Indeed, as shown on figure 2, the absence of any steady state would induce a decreasing dynamics of the relative price, such a decrease implying in turn that the price under which nobody is able to leave a bequest will be reached. The economy is frozen in an under-development trap, characterized by a nil capital stock. The following lemma states that having a sector of primary goods sufficiently efficient is a necessary condition for an economy to avoid under-development traps.

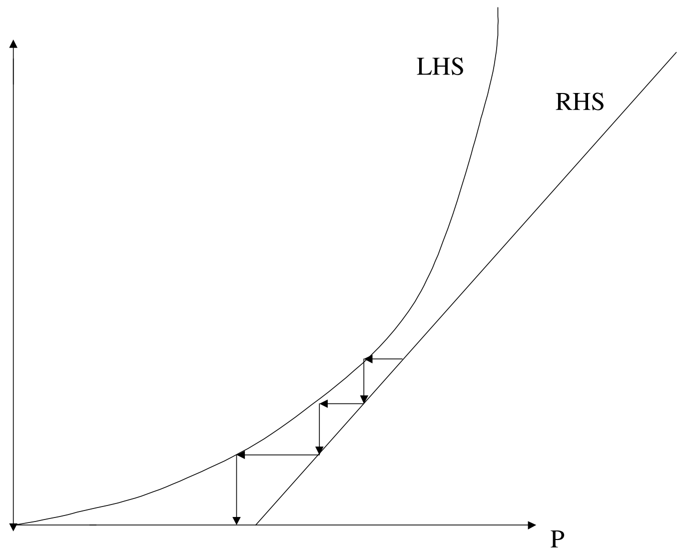

Figure 2

Lemma 3 $\forall \lambda > 0$, $\exists \hat{A}(\lambda)$ *such that if* $A > \hat{A}(\lambda)$, *there exist two steady states, one of which is stable.*

[8]First, because their wages are too low, and second, because such a low relative price implies a low amount of employment in the second sector. The high level of L_1 exerts a downward pressure on the rate of return (to allow the firm to reach its capital-labor ratio) and therefore this diminishes the income of rich individuals.

Proof. Through equation (19), when P goes to infinity, we have $LHS(P, A, \lambda) > RHS(P, \lambda)$. Moreover, $LHS(P, A, \lambda)$ is a decreasing function of A and its limit goes to zero when A tends to infinity. In contrast, $RHS(P, \lambda)$ is independent of A. This assures that with a sufficiently high A, there exists P such that $LHS(P, A, \lambda) < RHS(P, \lambda)$. This completes the proof. ■

This result is of course important for the description of the development process. More particularly, it will be of some interest to understand how the economy evolves endogenously from early to mature stages of development. The next section is devoted to the analysis of this process.

3 The Process of Development

For simplicity, let us assume that the initial bequest of poor individuals is nil *i.e.* $b_{-1}^P = 0$. Note that in the following sections, our inequality measure will be a simple ratio between the average income per rich and the average income per poor, that is to say[9] $Ineq_t = \frac{I_t^R}{I_t^P}$.

3.1 Presentation of both regimes :

As revealed by our analysis, if additional restrictions are further imposed on the basic model, the economy endogenously evolves through two distinct regimes :

Regime I :

In this early stage of development, the capital stock is low. As we have seen earlier, this implies a low production in sector 1. Since good 1 is retained as a *numéraire*, our regime is characterized by a low relative price. Moreover the low capital stock leads to a low marginal productivity of workers. Therefore (real) wages are too low and poor people are not able to consume other goods than primary goods. Rich people, because of a higher initial bequest, can satisfy their minimal consumption $\bar{C}_1$ of primary goods. Therefore, they can consume other goods and are especially able to leave a bequest to their offspring. This behavior implies an intergenerational wealth accumulation of the rich class.

Regime II :

[9] This index captures well the evolution of inequality. With this two groups' simple structure, a more complicated index would lead to the same inequality tendency. For instance, since the proportions of rich and poor are constant over time, the Lorenz curbe depends simply on the relative incomes.

At this mature stage of development, the capital stock is sufficiently high. Otherwise stated, it implies a sufficiently high (real) wage to allow the poorest to consume other goods, particularly to leave a bequest to their offspring. This implies an intergenerational wealth accumulation for both groups.

For each individual, there exists such a relative threshold price that, above this level, the individual will rationally choose to leave a bequest. Let us now characterize the threshold value between our 2 regimes.

Definition 2 *The threshold* $\tilde{P} = \frac{\bar{C}_1}{D}$ *is such that* $P > \tilde{P}$ *if and only if*[10] $I^P > \bar{C}_1$.

The position of $\tilde{P}$ will be crucial for the understanding of persistent inequality. If $\tilde{P}$ is too high, then rich people accumulation will not be sufficient for the relative price to increase above the threshold. As will be revealed from our subsequent argument, the steady state would then be an inegalitarian one, where poor stay poor and are constrained by the minimal consumption of primary goods.

3.2 Existence Conditions of Both Regimes

In this section, we study conditions under which our economy develops and evolves through both regimes. In section 3-5-1, we will remove some conditions and study their implications.

We need first, **(i)** steady state existence in both regimes **(ii)** price dynamics have to be increasing with time and such that $P_0 < \tilde{P}$ (the economy starts its development process in the first regime) and converges to a position where $P > \tilde{P}$ (i.e. in the second regime).

(i) Let $\hat{A}_\mu = \hat{A}(\mu)$ (respectively $\hat{A}_1 = \hat{A}(1)$) denotes the threshold productivity level such that a productivity level greater than this value ensures the existence of two steady states in the first (respectively second) regime (see lemma 3) where λ_t is equal to μ (respectively, to 1).

Assumption 1 : $A > max(\hat{A}_\mu, \hat{A}_1)$.

Definition 3 *Let* P_μ^S *and* P_μ^U *(respectively* P_1^S *and* P_1^U*) denote the two stable and unstable steady states in the first (respectively second) regime.*

[10] Recall that $b_{-1}^P = 0$.

Remark 1 *Through lemma* 1*, it is clear that*[11] $P_{\mu}^{S} < P_{1}^{S}$.

(ii) Then, we have to ensure that the sequence of equilibrium relative prices is increasing along the time path. This increase is a necessary condition for the poor to be able to bequeath to their offspring. Also, we require that our economy starts its development in the first regime and that it gradually enters in regime II.

Assumption 2: $P_{\mu}^{U} < P_0 < \tilde{P} < P_1^S$.

This assumption allows the economy to evolve through both regimes (*i.e.* to develop). Indeed, as the economy converges to P_1^S, it will access the second regime. In other words, the relative price will have sufficiently increased to allow poorest individuals to leave a bequest[12]. The next natural step consists in proving that assumptions 1 and 2 are compatible:

Lemma 4 *Assumptions* 1 *and* 2 *are compatible.*

Proof. The proof is provided in appendix 2. ■

Definition 4 *Let* $\tilde{t}$ *be such that* $\tilde{t} = Inf\{t \in \mathbb{N}^* \mid P_t \geq \tilde{P}\}$.

Then regime I corresponds to dates t such as $t < \tilde{t}$. In the same way, we can define regime II for every date such that $t \geq \tilde{t}$. Thus we can give values of the sequence $\{\lambda_t\}$:

$\lambda_t = \mu$ for $t < \tilde{t}$: referring to definition of the regime I , only rich individuals can bequeath.

$\lambda_t = 1$ for $t \geq \tilde{t}$: referring to the definition of regime II, rich and poor are able to bequeath.

3.3 Regime I

First, let us recall that at that stage, inequality at date t is measured by the following indicator: $Ineq_t = \frac{D+R_tK_t/\mu}{D}$, this regime being associated with values of t such that $t < \tilde{t}$.

In this regime, only rich individuals can leave a bequest. Therefore, the fraction of individuals leaving a bequest is equal to $\mu < 1$.

[11] Denote $G_\mu(.) = LHS^{-1} \circ RHS(.)$ (respectively $G_1(.)$) when $\lambda = \mu$ (respectively $\lambda = 1$). Then by lemma 1, $G_\mu(P) < G_1(P)$ if and only if $P > \frac{\bar{C}_1}{D}$, which is satisfied when we are in the second regime.

[12] Notice that if there exist steady states in the first regime then assumption 2 implies that there exist steady states in the second regime. This comes from the fact that $P^I < \tilde{P} < P^S$ implies $LHS(\tilde{P}, A, \mu) < RHS(\tilde{P}, A, \mu)$. But as both $LHS(\tilde{P}, A, \lambda)$ and $RHS(\tilde{P}, A, \lambda)$ are independent of λ, we can infer that $LHS(\tilde{P}, A, 1) < RHS(\tilde{P}, A, 1)$. But as was shown previously, when P goes to infinity, we have $LHS(P, A, 1) > RHS(P, 1)$. This shows the existence of steady states in the second regime.

Equilibrium dynamics in this first regime are described by :

$$\frac{[1-\mu(1-\beta)](DP_{t+1})^{1/\alpha}+\mu(1-\beta)\bar{C}_1(DP_{t+1})^{\frac{1-\alpha}{\alpha}}}{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{(\alpha}}} \tag{20}$$
$$= \beta\mu DP_t-\beta\mu\bar{C}_1+\frac{\beta\alpha}{1-\alpha\beta}\{[1-\mu(1-\beta)]DP_t+\mu(1-\beta)\bar{C}_1\}$$

Proposition 1 *Under assumptions* 1 *and* 2*, the early stage in the development process is characterized by a decrease of the labor factor in the sector producing primary goods, an increase in inequality.*

Proof. The proof is straightforward since demand in good 2 increases, and since this good is produced uniquely with labor, the labor force must increase in that sector and thus decrease in sector 1. In addition, in this regime only rich people bequeath to their offspring. Moreover, their bequests rise along the equilibrium path (*i.e.* the relative price increases) generating an increase in inheritance inequalities. ■

This continuous increase in the bequests of rich individuals implies a rise in marginal productivity of workers. As we have shown before, this brings about an increase in the relative price. This period is thus characterized by a continuous rise in the real wealth of poor individuals. This will eventually allow them to consume their threshold of primary goods and leave a bequest. In this case, the economy has entered in the second regime[13].

3.4 Regime II

This regime is associated with the dates $t > \tilde{t}$. In this regime, all individuals, rich or poor, can afford the threshold of primary goods consumption and thus can bequest. Therefore, the poor bequeath and start a process of intergenerational wealth accumulation.

The dynamics of this regime ($\lambda_t = 1\ \forall t > \tilde{t}$) states as:

$$\frac{\beta(DP_{t+1})^{1/\alpha}+(1-\beta)\bar{C}_1(DP_{t+1})^{\frac{1-\alpha}{\alpha}}}{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{(\alpha}}} \tag{21}$$
$$= \beta DP_t-\beta\bar{C}_1+\frac{\beta\alpha}{1-\alpha\beta}\{\beta DP_t+(1-\beta)\bar{C}_1\}$$

[13] To be precise, at date $\tilde{t}$ there exists an intermediary regime between regimes I and II. This regime is characterized by the dynamics described by equation (19) with $\lambda_{\tilde{t}+1} = 1$ and $\lambda_{\tilde{t}} = \mu$. This is to say, the poor leaving a bequest in $\tilde{t}$ did not earn any bequest from their parents.

Proposition 2 *Under assumptions* 1 *and* 2*, at the beginning of this regime, an important reallocation from the sector of primary goods to the sector of luxury goods occur and aggregate savings increases dramatically. Moreover, a decline in the inheritance inequality occurs in this regime.*

Proof. The labor force in sector 2 jumps at a higher level at the beginning of this regime since poor individuals are able to consume luxury goods (that is produced uniquely with labor). Moreover, poor individuals are also able to leave bequest, therefore this explains the jump in the level of aggregate savings.

Under assumptions 1 and 2, we can show that both steady states are egalitarian in this second regime : we know that the relative price reaches a steady state denoted by $\bar{P}$. $\bar{R}$ denotes the rate of return at the steady state. Let b^R and b^P respectively denote the bequest of the rich and of the poor at the steady state, both being determined by the same equation :

$$b^R = \beta[D\bar{P} + b^R\bar{R} - \bar{C}_1]$$

$$b^P = \beta[D\bar{P} + b^P\bar{R} - \bar{C}_1].$$

This implies $b^R = b^P$. The stable steady state of this regime being characterized by an egalitarian situation, we can infer that wealth converges. This regime is characterized by wealth accumulation of both classes. Then there exists a date $t \in [\tilde{t}, +\infty[$ from which poor people increase their bequest - from one date to another - more than the rich do. Consequently, inheritance inequality will start to decrease along the time path. The absence of market imperfection, added to the *ex-ante* homogeneity of individuals, implies that, in the long run, the level of real wealth of the poor individuals converges to that of the rich. Hence, in this case (characterized by a sufficiently large increase in relative price), long-run equilibrium corresponds to an egalitarian steady state.
■

Note however that though the reversal in inequalities evolution occurs in this regime, it does not coincide with the date $\tilde{t}$ of entrance in this regime. Moreover, in conformity with evidences, this regime is characterized by a rise in productivity of labor in the sector producing primary goods (agricultural and manufactured goods). In this sector, labor decreases whereas it increases in the sector producing luxury goods (or services). This reproduces the movements in the labor force over the last centuries in most developing countries[14]. Then our model is consistent with the massive sectorial labor reallocation experienced in the U.S. and in most growing economies during the last century.

3.5 Persistent Inequality and the Effect of Redistribution

3.5.1 Under-development and Poverty Traps

Inequality persistency as well as development are conditional to assumptions 1 and 2. But, as we have seen in previous sections, under-development and poverty traps can exist. Under-development traps can occur when the dynamics of the relative price is decreasing (for example, when the initial stock of capital is not sufficient, that implies $P_0 < P_\mu^U$, or when a steady state does not exist *i.e.* A is too low). Indeed, this implies that rich individuals become poorer and poorer and after a certain date, they will not be able to bequeath, the economy ending without any physical capital. Poverty traps occur when the increase in rich individuals' accumulation is too low and the relative price does not increase anymore. In other words, under certain conditions, a country can be durably stuck in the early stages of industrialization. It is likely that the economy reaches its steady state too early, so that the threshold value of relative price (allowing the poorest to leave a bequest) will not be reached (see figure 3 for a graphical illustration). Such a phenomenon can appear under parameters conditions such as $P_\mu^S < \tilde{P}$. This case occurs when μ is too low *i.e.* when sufficient (initial) inequality does not exist (this last point will be developed in the next section).

Persistent inequality is possible in our model because dynamics governing bequest evolution is distinct between rich and poor individuals. Loury (1981)'s results are effective as soon as our economy enters in the second regime, *i.e.* a long-run egalitarian distribution and above all independent of the initial distribution. As a consequence, when the relative price increase is not sufficient, poor people will not bequeath to their offspring and the economy is stuck into a poverty trap (*i.e.* inequality reversal will not occur).

[14]This result is quite intuitive : the aggregate demand for goods 1 becomes constant and equal to $\bar{C}_1$. Then (at equilibrium) an increase in the capital stock necessarily implies a reallocation of the labor force from sector 1 to sector 2.

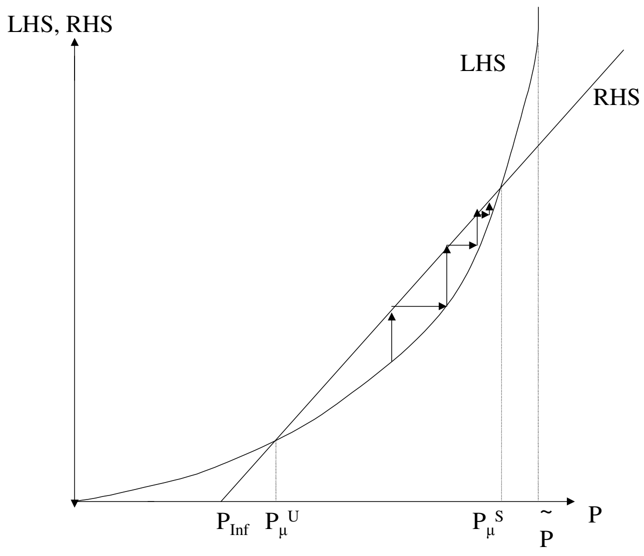

Figure 3

3.5.2 The Impact of Redistribution:

First, notice that redistribution in the second stage will not have any impact on aggregate variables since poor individuals and rich individuals have the same behavior and noticeably, the same marginal propensity to save. So, in the sequel, we study the impact of redistribution when there exist some individuals who are unable to save (that is to say in the first regime).

In this framework, two qualitatively different impacts can happen regarding the type of redistribution (in the sequel, we denote a variable x after redistribution with $\hat{x}$). First, imagine a redistribution which transfers wealth from rich to poor through initial bequests at the beginning of times (*i.e.* $\hat{b}_{-1}^R = b_{-1}^R - \tau$ and $\hat{b}_{-1}^P = b_{-1}^P + \frac{\mu}{1-\mu}\tau = 0 + \frac{\mu}{1-\mu}\tau$). We assume that τ is sufficiently small in order to keep constant the number of individuals being able to save and to bequeath. Obviously, such a policy will diminish aggregate capital stock at date 1 because it channels resources towards individuals who do not save. Formally, the relative price dynamics changes between date 0 and 1 : $LHS(P_1) = RHS(P_0) - \mu\tau\beta[\frac{1-\alpha}{(1-\alpha\beta)}]\alpha A[A(1-\alpha)]^{\frac{1-\alpha}{\alpha}}[DP_0]^{\frac{\alpha-1}{\alpha}}$. For the following periods we have exactly the same dynamics of the capital stock with lower "initial"(at date 1) conditions. Then capital stock is lower at each date but it converges towards the same steady state. As physical capital is the only engine of our economy, its decrease leads to diminutions of both outputs (beginning at date 1)[15]. More surprisingly, under assumptions 1 and 2, this policy implies that the beginning of the inequality decrease will be delayed. Indeed, this is quite intuitive since we need a longer accumulation, in order to reach a sufficient real wealth level to break away from the minimal consumption threshold ($\hat{P}_0 < P_0 < \tilde{P}$)[16]. Then such a redistribution clearly starts by decreasing the level of inequality (since at the beginning of times, rich individuals have less wealth) but this diminution vanishes and inequality persists for a longer time. To go further, one can easily check that such a redistribution can freeze the economy in an under-development trap whereas this would not happen if the redistribution did not occur. Otherwise stated, assumptions 1 and 2 can be satisfied before the intervention of a redistribution scheme, not after (*i.e.* $\hat{P}_0 < P_\mu^U < P_0$).

The other way to redistribute consists in increasing the number of individuals being able to bequeath, keeping constant the level of capital stock. Notice that such a redistribution is not always feasible since a redistribution does not always give the possibility to escape from the minimal consumption threshold[17]. We assume that circumstances are such that redistribution gives to poor individuals the ability to consume more than $\bar{C}_1$. One could think of redistributions toward middle class individuals allowing them to start an intergenerational wealth accumulation. To implement such a redistribution, the policy reduces the initial bequest of rich individuals by τ and then transfers it towards a fraction ε of the poor in order to provide the same wealth to every individual who is able to save (then one can check that $\varepsilon = \frac{\tau\mu}{b_{-1}^R - \tau}$). Such a policy leads to an increase in the number of individuals being able to bequeath and to a decrease in the level of bequest per rich individual and thus leads to a reduction in the level of inequality. Main differences from the previous case relies on the fact that the dynamics changes. Formally, we know by lemma 1 that $P_{t+1} = G(\mu, P_t)$[18] is decreasing in μ. As initial conditions do not change, this implies that the relative price dynamics is lower. This is due to the fact that at each date, more individuals consume the threshold of primary goods.

[15] This kind of argument is rather old since it was suggested first by Adam Smith (1776) and was further interpreted and developped by Keynes (1920), Lewis(1954), Kaldor (1955), Bourguignon (1981), and Galor and Moav (2004).

[16] Remember that there exists an increasing (static) relationship between capital stock and relative price.

[17] A slight modification of our model with a continuum of wealth levels would lead a redistribution to the median class to systematicaly increase the number of people able to consume their minimal consumption threshold.

[18] As noted previously, $G(P, \mu) \equiv LHS^{-1} \circ RHS(P, \mu)$.

To be more precise, note μ and $\hat{\mu}$ the proportion of individuals being able to bequeath respectively before and after the redistribution. By assumption, $\hat{\mu} > \mu$ ($\hat{\mu} - \mu = \varepsilon > 0$), $K_0 = \mu b^R_{-1}$ and $K_0 = \hat{K}_0 = \hat{\mu}\hat{b}^R_{-1}$.

Without redistribution, the capital stock at date 1 is

$$K_1 = \mu b^R_0 = \mu\beta[DP_0 + b^R_{-1}R_0 - \bar{C}_1] = \mu\beta[DP_0 - \bar{C}_1] + \beta K_0 R_0.$$

While after redistribution, the capital stock at date 1 is

$$\hat{K}_1 = \hat{\mu}\hat{b}^R_0 = \hat{\mu}\beta[D\hat{P}_0 + \hat{b}^R_{-1}\hat{R}_0 - \bar{C}_1].$$

But since by assumption, $K_0 = \hat{K}_0$ (and so $P_0 = \hat{P}_0$ and $R_0 = \hat{R}_0$), one can show that

$$\hat{K}_1 = (\hat{\mu} - \mu)\beta[DP_0 - \bar{C}_1] + \mu\beta[DP_0 - \bar{C}_1] + \beta K_0 R_0 = (\hat{\mu} - \mu)\beta[DP_0 - \bar{C}_1] + K_1.$$

Then $\hat{K}_1 < K_1 \Leftrightarrow DP_0 < \bar{C}_1$. But as specified earlier, our redistribution takes place during the first regime, therefore we know that $P_0 < \tilde{P} = \frac{\bar{C}_1}{D}$. This proves that $\hat{K}_1 < K_1$. The aggregate wealth of rich individuals is indeed the same but since they are more, they devote a bigger part of their aggregate wealth to the consumption of the threshold of primary goods. This is why $\hat{K}_1 < K_1$. Since this reasoning can be done for each time period, this in turn implies that the level of aggregate capital stock diminishes at every date. Thus, it is more likely for the economy to be frozen in an under-development trap. As in the previous case, for an economy satisfying assumption 1 and 2, inequality reversal is delayed[19].

The study therefore suggests that an economy that prematurely implements a policy designed to enhance equality may be trapped at a low stage of development. An under-development economy, which values equality as well as prosperity, may confront a trade-off between equality and stagnation in the long-run and inequality in the short-run followed by equality and prosperity in the long-run[20]. Nonetheless, this trade-off is far from being inevitable since there exists other policy instruments. For instance, the government could use his savings to speed accumulation and hence structural change. More precisely, it is clear that the marginal propensity to save

[19] To enhance growth in such a framework, the best redistribution policy would consist in channelling the entire wealth towards a single individual with a nil mass. This in turn reduces the amount of aggregate consumption of primary goods and allows for more saving.

[20] This point has been underlined for instance in Galor and Tsiddon (1997) with a very different theoretical argument.

of the government is equal to one, whereas the one of individuals is always strictly less than 1. Therefore a policy consisting in increasing government savings at the beginning of times (through for instance taxes on rich individuals if assumptions 1 and 2 are satisfied) would increase the capital stock at each date and thus this policy implies that the beginning of the inequality decrease will arrive sooner. Then such a policy decreases the level of inequality and inequality persists for a shorter time. Moreover, this policy would also allow an economy to get off under-development traps. Interestingly, this kind of policy would be efficient in both regimes.

A Appendix 1

We first give the details to obtain equation (18). Then we derive equation (19).

Through factor price frontier of firm 2 (4), under the optimality condition (13), and the luxury goods equilibrium (16), we have[21]

$$\begin{aligned} L_{2,t} &= (1/D)\int_0^1 C^i_{2,t}di \\ &= (1/D)(1-\beta)\{\lambda_t[D-\frac{\bar{C}_1}{P_t}]+\frac{K_tR_t}{P_t}\}. \end{aligned} \tag{22}$$

Together with the factor price frontier of firm 2 (4), optimal capital-labor ratio (9), and the labor market equilibrium equation (17), we obtain:

$$L_{1,t}=\frac{1-\alpha}{1-\alpha\beta}[1-\lambda_t(1-\beta)+\frac{\lambda_t(1-\beta)\bar{C}_1}{DP_t}]. \tag{23}$$

Then using the factor price frontier in firm 1 (8) with the demand of capital-labor ratio in firm 1 (9), we get:

$$\{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{\alpha}}\}K_t=[1-\lambda_t(1-\beta)](DP_t)^{1/\alpha}+\lambda_t(1-\beta)\bar{C}_1(DP_t)^{\frac{1-a}{\alpha}}. \tag{24}$$

Therefore, integrating equation (14)[22] (defining bequest evolution between generations) on the set of individuals, and using again equations (4), (9), (23) and (18), we get the following equation for equilibrium dynamics :

[21] As the reader will have noticed, we implicitly assume that the sequence of λ_t is such that $\lambda_t \geq \mu$. This will be satisfied at equilibrium.

[22] Notice that aggregation is particularly easy in this model. The linearity of the equation defining bequest implies that aggregate wealth does not depend on relative wealth. Indeed, wealth can only differ between individuals through different initial bequests. This is why we can solve dynamics through a unique equation in K_{t+1} (and alternatively in P_{t+1}) which can be considered as the bequest of a representative agent having the same properties as a typical agent of our model.

$$\frac{[1-\lambda_{t+1}(1-\beta)](DP_{t+1})^{1/\alpha}+\lambda_{t+1}(1-\beta)\bar{C}_1(DP_{t+1})^{\frac{1-\alpha}{\alpha}}}{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{(1-\alpha)}{(\alpha}}} \tag{25}$$

$$= \beta\lambda_t DP_t - \beta\lambda_t\bar{C}_1 + \frac{\beta\alpha}{1-\alpha\beta}\{[1-\lambda_t(1-\beta)]DP_t + \lambda_t(1-\beta)\bar{C}_1\}.$$

B Appendix 2

In this part, we prove lemma 4.

First, the value of the initial relative price depends on the initial conditions on bequest. This is due to the fact that, first there exists a static relationship between K_t and P_t given by equation (18). And second, K_t is given by the sum of every initial bequests (see equation (15)). Thus, we assume initial conditions on b_{-1}^R such that $P^I < P_0 < \tilde{P}$. Let us study conditions upon which the other part of assumption 2 is based: a necessary and sufficient condition to have $P^I < \tilde{P} < P^S$ is that $RHS(\tilde{P},\mu) > LHS(\tilde{P},A,\mu)$ (see figure 1). One can prove that $RHS(\tilde{P},\mu) = \frac{\beta\alpha\bar{C}_1}{1-\alpha\beta}$ and that $LHS(\tilde{P},A,\mu) = \frac{(\bar{C}_1)^{1/\alpha}}{(1-\alpha\beta)A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}}}$. Then, if $A > \frac{(\bar{C}_1)^{1-\alpha}}{[\beta\alpha(1-\alpha)]^\alpha}$, the inequality $RHS(\tilde{P},\mu) > LHS(\tilde{P},A,\mu)$ is satisfied. Thus with an adequate choice in A (such that $A > Max\{\frac{(\bar{C}_1)^{1-\alpha}}{[\beta\alpha(1-\alpha)]^\alpha}, Max(\hat{A}_\mu,\hat{A}_1)\}$), both assumptions are solved. This completes the proof.

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