European Economic Review 31 (1987) 417–426. North-Holland

IMPERFECT COMPETITION, UNEMPLOYMENT AND POLICY

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1. Introduction

The object of this paper is to construct a simple general equilibrium model with imperfect competition, displaying endogenous price making resulting from explicit utility or profit maximization by agents, and to study within this framework both some microeconomic problems, such as the efficiency properties of the equilibrium obtained, or some macroeconomic ones such as the existence of unemployment or the effectiveness of some governmental monetary policies. This paper is thus very much in line with earlier models linking imperfect competition and equilibria with nonclearing markets, a tradition initiated by Benassy (1976, 1977, 1982) and Negishi (1977, 1979), and continued by Hahn (1978), Hart (1982), Snower (1983), Weitzman (1985), among others.

2. The model

We shall consider a monetary economy with three types of goods: Money, different types of labor indexed by $i=1,\dots,n$, and consumption goods indexed by $j=1,\dots,m$. There are three types of agents: Households indexed by $i=1,\dots,n$, firms indexed by $j=1,\dots,m$, and government. Consumer i is the only one to be endowed by labor of type i, firm j is the only one to produce good j. We shall call w_i the money wage for type i labor, p_j the price of good j, w and p the corresponding vectors

$$p=\{p_j|j=1,\dots,m\}, \qquad w=\{w_i|i=1,\dots,n\}.$$

Firm j produces a quantity of output y_j according to a production function $y_j=F_j(l_j)$ where l_j is a vector with components $l_{ij}, i=1,\dots,n$, and l_{ij}

*This is an abridged version of a paper [Benassy (1986b)] presented at the session 'Theories of Involuntary Unemployment', EEA Congress, Vienna, August 1986. I wish to thank D. Snower for his comments on this first draft.

is the quantity of labor i used by firm j. We shall assume F_j strictly concave in its arguments. Firm j maximizes its profits $\pi_j = p_j y_j - w l_j$.

Household i has initial endowments $\bar{l}_i$ of type i labor, and $\bar{m}_i$ of money. It consumes a vector c_i with components c_{ij}, $j = 1, \ldots, m$, sells a quantity of labor l_i and ends up with a quantity of money m_i. Call θ_{ij} the share of firm j owned by household i. The budget constraint is

$$pc_i + m_i = w_i l_i + \mu \bar{m}_i + \sum_{j=1}^{m} \theta_{ij} \pi_j,$$

where μ is the policy parameter of the government, which is assumed to be able to increase proportionately all households' money holdings. This particular policy has been chosen because it is known to be 'neutral' in Walrasian equilibrium, which will thus allow an easy comparison.

Household i maximizes a utility function $U_i(c_i, l_i, m_i, \mu)$ which is assumed to be strictly quasi-concave in c_i, $-l_i$ and m_i. The arguments m_i and μ represent the indirect utility of money,[1] and we shall assume that U_i is homogeneous of degree zero in m_i and μ. The implicit idea is that this indirect utility is homogeneous of degree zero in m_i and future prices, and, other things equal, these future prices are expected to move proportionately to μ (a property which, as we shall see in section 5, is indeed verified in the current period).

In this model each good (labor or consumer good) is sold by a single agent to a multitude of other agents. We shall assume, as is traditional in such models, that the price is decided upon by the single seller. So firm j sets price p_j, household i sets wage w_i. We shall assume that they do so using objective demand curves [Gabszewicz and Vial (1972), Nikaido (1975), Benassy (1986a)]. The equilibrium is a Nash equilibrium in prices and wages, conditionally on these objective demand curves.

3. Equilibrium: Definition and characterization

Each seller sells only one good, and, as we shall see below, sets its price high enough so as to be willing to satisfy all demand for that good. In equilibrium each agent will thus be constrained only on his sales. We shall denote by $L_i(p, w, \mu)$ and $Y_j(p, w, \mu)$ the objective demand for labor i and output j respectively, i.e., more precisely the quantity constraint on sales faced by household i on labor market i, or by firm j in output market j. These functions are derived in the appendix. We should only note at this stage that $L_i(p, w, \mu)$ and $Y_j(p, w, \mu)$ are homogeneous of degree zero in their arguments.

[1]See Grandmont (1983) for the general methodology in the Walrasian case, and Benassy (1982) in the non-Walrasian one.

We can now derive the optimal prices and transactions of the agents. Consider first firm j; it will solve the following profit maximization program ($\mathrm{A_j}$):

$$\text{Maximize } p_j y_j - w l_j \qquad \text{s.t.}$$

$$\begin{cases} y_j = F_j(l_j), \\ y_j \leqq Y_j(p, w, \mu). \end{cases} \tag{$\mathrm{A_j}$}$$

We assume this program has a unique solution, which thus yields optimal price p_j and the optimal production plan (y_j, l_j) as functions of government policy and the other price variables

$$p_j = \psi_j(p_{-j}, w, \mu), \qquad (y_j, l_j) = \phi_j(p_{-j}, w, \mu),$$

where p_{-j} is the vector of all prices, except p_j.

Consider now household i. It chooses the wage w_i, labor sales l_i and consumption vector c_i so as to maximize utility according to the following program ($\mathrm{A_i}$):[2]

$$\text{Maximize } U_i(c_i, l_i, m_i, \mu) \qquad \text{s.t.}$$

$$\begin{cases} pc_i + m_i = \mu \bar{m}_i + w_i l_i + \sum_{j=1}^{m} \theta_{ij} \Pi_j(p, w, \mu), \\ l_i \leqq L_i(p, w, \mu), \\ l_i \leqq \bar{l}_i, \end{cases} \tag{$\mathrm{A_i}$}$$

which yields the functions

$$w_i = \psi_i(w_{-i}, p, \mu), \qquad (l_i, c_i) = \phi_i(w_{-i}, p, \mu),$$

where w_{-i} is the vector of all wages, except w_i. We can now define our equilibrium concept.

Definition. An imperfectly competitive equilibrium is a set of prices p_j^*, wages w_i^*, outputs y_j^*, labor sales l_i^*, consumption vectors c_i^*, labor input vectors l_j^* such that:

(a) $w_i^* = \psi_i(w_{-i}^*, p^*, \mu), \quad \forall i,$
(b) $p_j^* = \psi_j(p_{-j}^*, w^*, \mu), \quad \forall j,$

[2] $\Pi_j(p, w, \mu)$ is the profit associated with the triple (p, w, μ); see the appendix.

(c) $(l_i^*, c_i^*) = \phi_i(w_{-i}^*, p^*, \mu), \quad \forall i,$
(d) $(y_j^*, l_j^*) = \phi_j(p_{-j}^*, w^*, \mu), \quad \forall j.$

Note that we have omitted in this definition the traditional equations stating the equality of purchases and sales in each market. Indeed, in view of the definition of the objective demand curves, which is based on a fixprice equilibrium notion, the consistency of transactions is automatically assured.

Of course the equilibrium will change as μ changes. In what follows we shall assume that to a given government policy is associated a unique equilibrium. Before studying its various properties, we shall characterize it by deriving a number of partial derivatives.

Consider first the program (A_j) above yielding the optimal actions of firm j. The Kuhn–Tucker conditions associated to this program yield immediately, assuming an interior solution

$$\frac{\partial F_j}{\partial l_{ij}} = \frac{w_i}{p_j} \frac{1}{(1 - 1/\varepsilon_j)}, \tag{1}$$

where $\varepsilon_j = -(p_j/y_j)\,\partial Y_j/\partial p_j$ is the absolute value of the own price elasticity of objective demand for good j. At an equilibrium ε_j is greater than 1. These conditions can also be rewritten, using firm j's cost function $\chi_j(y_j, w)$,

$$\frac{\partial \chi_j}{\partial y_j} = p_j\left(1 - \frac{1}{\varepsilon_j}\right), \tag{2}$$

which is the traditional 'marginal cost equals marginal revenue' equation. Let us turn now to the program (A_i) yielding agent i's optimal actions. Assume an interior solution, and in particular that the last constraint is not binding. Call λ_i the 'marginal utility' of wealth, i.e., the Kuhn–Tucker multiplier of the budget constraint. We obtain the following conditions:[3]

$$\frac{\partial U_i}{\partial m_i} = \lambda_i, \qquad \frac{\partial U_i}{\partial c_{ij}} = \lambda_i p_j, \tag{3}$$

$$\frac{\partial U_i}{\partial l_i} = -\lambda_i w_i\left(1 - \frac{1}{\varepsilon_i}\right), \tag{4}$$

where $\varepsilon_i = -(w_i/l_i)\,\partial L_i/\partial w_i$. Again at equilibrium $\varepsilon_i > 1$.

With the help of these differential characterizations, we can now describe some salient properties of our equilibrium.

[3]We actually assume to simplify that the influence of w_i on household i's profit income is negligible, which will be the case if there are many households, and each owns negligible shares of each firm.

4. Underemployment, underproduction and inefficiency

We shall now show that, even though prices are fully flexible and rationally decided upon by agents, the equilibrium allocation has properties which strongly differentiate it from those of a Walrasian equilibrium.

First we see that at equilibrium there is both underemployment and underproduction. Indeed eqs. (1) and (2) show that at the going prices and wages firm j would be happy to produce and sell more, if the demand was forthcoming, thus displaying *underproduction.* Symmetrically eq. (4) shows that the household i would like to sell more of its labor if the demand was there, thus displaying *underemployment.*

We shall further show that employment and production are inefficiently low in the following strong sense: it is possible to find increases in production and employment which would increase both firms' profits and households' utilities at the equilibrium prices and wages. This is made precise in the following proposition.

Proposition 1. Consider at the given price and wage system p^* and w^* some small increases in production $\mathrm{d}y_j > 0$, $j = 1, \ldots, m$, and the associated cost minimizing increases in employment. If extra production is distributed in a way which respects the corresponding additional incomes of households, the new allocation yields higher profits for all firms and higher utilities for all households.

Proof. Consider thus a set of $\mathrm{d}y_j > 0$, $j = 1, \ldots, m$. Each firm produces this extra production efficiently, which yields extra employment $\mathrm{d}l_i > 0$, $i = 1, \ldots, n$. Consider first the profit variation for firm j

$$\mathrm{d}\pi_j = p_j\,\mathrm{d}y_j - (\partial \chi_j / \partial y_j)\,\mathrm{d}y_j,$$

and in view of eq. (2)

$$\mathrm{d}\pi_j = \frac{p_j\,\mathrm{d}y_j}{\varepsilon_j} > 0. \tag{5}$$

Turn now to household i. The value of extra consumptions must sum up to the value of extra incomes

$$\sum_{j=1}^{m} p_j\,\mathrm{d}c_{ij} = w_i\,\mathrm{d}l_i + \sum_{j=1}^{m} \theta_{ij}\,\mathrm{d}\pi_j. \tag{6}$$

The increment in utility is, since m_i and μ do not change

$$\mathrm{d}U_i = \sum_{j=1}^{m} (\partial U_i / \partial c_{ij})\,\mathrm{d}c_{ij} + (\partial U_i / \partial l_i)\,\mathrm{d}l_i,$$

which, using eqs. (3)–(6) becomes

$$\mathrm{d}U_i=\lambda_i\left[\frac{w_i\,\mathrm{d}l_i}{\varepsilon_i}+\sum_{j=1}^{m}\frac{\theta_{ij}p_j\,\mathrm{d}y_j}{\varepsilon_j}\right]>0. \qquad \text{Q.E.D.} \tag{7}$$

The above proposition shows in a constructive way, by exhibiting some adequate incremental trades, that the imperfectly competitive equilibrium is quite inefficient. Indeed the above inefficiency results are even stronger than Pareto inefficiency, since we have constrained the incremental trades to be consistent with the equilibrium price wage system, whereas such a constraint is not required to prove Pareto inefficiency. We should also note that these inefficiencies are quite similar to those observed in 'Keynesian type' general excess supply states [see for example Benassy (1977, 1982)].

5. Neutrality of monetary policy

We shall now show that, in spite of its 'Keynesian' characteristics, the system described above reacts to monetary policy in a more 'Walrasian' than 'Keynesian' manner. Namely, monetary policies taking the form of proportional increases in initial money holdings are ineffective, or 'neutral', just as in Walrasian models.

Proposition 2. Consider a proportional increase in all initial money holdings (i.e., $\mu>1$). Then production, employment and utilities do not change. Prices, wages and profits are multiplied by μ.

Proof. The proof of that result is quite trivial if one looks at the programs (A_j) and (A_i) yielding the optimal actions of agents; indeed from these programs it is clear that

$$\psi_j(\mu p_{-j},\mu w,\mu)=\mu\psi_j(p_{-j},w,1), \qquad \phi_j(\mu p_{-j},\mu w,\mu)=\phi_j(p_{-j},w,1),$$

$$\psi_i(\mu w_{-i},\mu p,\mu)=\mu\psi_i(w_{-i},p,1), \qquad \phi_i(\mu w_{-i},\mu p,\mu)=\phi_i(w_{-i},p,1).$$

In view of these homogeneity properties, y_j^*, l_j^*, l_i^*, c_i^* are homogeneous of degree 0 in μ, whereas p^* and w^* are homogeneous of degree one in μ. Q.E.D.

6. Efficient policies

Clearly the problem in our equilibrium is that, because of the Nash equilibrium structure, agents set prices and wages 'too high'. The resulting spillovers generate the inefficiency. Intuition thus suggests that monetary policy, together with some general incomes policy which would decouple price and wage inflation from money growth, could be effective. We shall again constructively prove this by considering the simple example of a price-wage freeze coupled with a small monetary expansion $\mathrm{d}\mu > 0$.[4] To carry out the exercise we shall make the further assumption of 'stabilized money holdings', i.e., that at the equilibrium considered $m_i^* = \bar{m}_i$. Such will be the case if the economy is in a state of long-run equilibrium to start with. This also contains as a particular case the situation, most often considered in the literature, where the model is symmetrical.

Proposition 3. Consider an equilibrium with stabilized money holdings, and a proportional increase $\mathrm{d}\mu$ *in money holdings, prices and wages being maintained constant. This will increase all firms' profits and all households' utilities.*

Proof. Let us consider, as in Proposition 1, positive increases in production levels $\mathrm{d}y_j > 0$ and the associated cost minimizing increases in employment $\mathrm{d}l_i > 0$. It is easy to show that under the assumption of stabilized money holdings formulas (5) and (7) above still hold. Now, however, the increases $\mathrm{d}y_j$ and $\mathrm{d}l_i$ are not arbitrary, but must result from voluntary trading. So let us first note that in view of eqs. (2) and (4), the firms will be willing to make small expansions of production and the households will willingly increase their labor sales if the demand is forthcoming. Secondly one can show [Benassy (1986b, appendix 2)] that, assuming that goods and money are 'normal', such increases in y_j and l_i can actually be engineered at fixed wages and prices via the monetary increase $\mathrm{d}\mu$ through traditional multiplier effects, yielding increases of the form

$$p_j \,\mathrm{d}y_j = k_j \,\mathrm{d}\mu, \qquad k_j > 0,$$

$$w_i \,\mathrm{d}l_i = k_i \,\mathrm{d}\mu, \qquad k_i > 0.$$

Feeding these into eqs. (5) and (7), we find

$$\mathrm{d}\pi_j = \frac{k_j}{\varepsilon_j} \,\mathrm{d}\mu > 0,$$

$$\mathrm{d}U_i = \lambda_i \left[\frac{k_i}{\varepsilon_i} + \sum_{j=1}^{m} \theta_{ij} \frac{k_j}{\varepsilon_j} \right] \mathrm{d}\mu > 0. \qquad \text{Q.E.D.}$$

We can now note that the reaction of our equilibrium to the policy

[4] A proportional decrease in prices and wages with constant money holdings would of course, because of the homogeneity properties, have similar results.

considered in Proposition 3 is very different from the reaction of a Walrasian system, where there would be no additional employment and production following the same policy.

7. Conclusions

We have seen here a general equilibrium model with endogenous price and wage making by agents internal to the system. A particularly interesting feature of the corresponding equilibrium are the inefficiencies generated by the specific Nash structure in prices and wages and the intermarket spill-overs (Proposition 1). Even though these inefficiencies are very akin to Keynesian type ones, we found that monetary policy is ineffective and 'neutral', as in Walrasian models (Proposition 2).

A worthwhile topic of research is to investigate policies, or combinations of policies, which do not have these neutrality properties. As an example we exhibited a very simple combination of monetary and 'incomes' policies which could lead to Pareto improvements (Proposition 3). Of course more sophisticated (and less interventionist) policies should be an interesting topic of research.

In this paper we primarily emphasized the aspects arising from the general equilibrium nature of the problem, taking for granted the 'degrees of market power' of the agents subsumed in the parameters ε_i and ε_j. In order to be able to propose effective policy measures, it seems now necessary to assess, in partial or general equilibrium settings, the deep causes for which the functioning of each market deviates from a competitive one, a subject which is taken up by some of the companion papers to this one.

Appendix

We shall in this appendix show how to compute the objective demand curve in the zone of 'general excess supply', which is the zone the equilibrium lies in [the general method for dealing with objective demand curves is given in Benassy (1986a)]. We shall take any p, w and μ corresponding to this zone, and find out which demands for goods and labor types will arise once all feedback effects have been taken into account. Intuitively this boils down to finding total demand for goods i and j at a fixprice equilibrium corresponding to p, w and μ.

For given (p, w, μ) firm j is constrained on its sales of output j. It thus solves the following program:

$$\text{Maximize } p_j y_j - w l_j,$$

$$y_j \leqq F_j(l_j),$$

where y_j is a binding constraint exogenous to the firm. The solution is a set of labor demands $L_{ij}(y_j, w)$, and a cost function $\chi_j(y_j, w)$.

Similarly household i solves the following program:

$$\text{Maximize } U_i(c_i, l_i, m_i, \mu) \qquad \text{s.t.}$$

$$pc_i + m_i = \mu\bar{m}_i + w_i l_i + \sum_{j=1}^{m} \theta_{ij}\pi_j,$$

where p, w, l_i and the π_j's are given. The solution is a set of consumption demands $C_{ij}(\mu\bar{m}_i + w_i l_i + \sum_{j=1}^{m} \theta_{ij}\pi_j, l_i, p, \mu)$.

Now consider the following mapping:

$$y_j \to \sum_{i=1}^{n} C_{ij}\left(\mu\bar{m}_i + w_i l_i + \sum_{j=1}^{m} \theta_{ij}\pi_j, l_i, p, \mu\right),$$

$$l_i \to \sum_{j=1}^{m} L_{ij}(y_j, w),$$

$$\pi_j \to p_j y_j - \chi_j(y_j, w).$$

Assuming a unique fixed point, this yields functions $Y_j(p, w, \mu)$, $L_i(p, w, \mu)$ and $\Pi_j(p, w, \mu)$ which represent respectively the objective demand for good j, labor i, and the associated profits of firm j.

Secondly, let us note that the functions L_{ij} are homogeneous of degree zero in w, the functions C_{ij} are homogeneous of degree zero in μ, w_i, p, and π_j, and profits themselves are homogeneous of degree one in p_j and w. From that we deduce Y_j and L_i are homogeneous of degree zero, and Π_j homogeneous of degree one, in the arguments p, w and μ.

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