Chapter 37

MONOPOLISTIC COMPETITION

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Contents

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1. Introduction

Monopolistic competition, a term coined in the famous contribution of Chamberlin (1933),[1] is usually defined as a situation of imperfect competition with the following features: (a) the products sold are differentiated; (b) firms themselves set the price of these goods; (c) the number of sellers is large and each firm disregards the effects of its price decisions on the actions of its competitors; (d) entry is unrestricted and proceeds until profits are reduced to zero, or the smallest possible number consistent with the fact that the number of firms is an integer.

What has been called by Samuelson (1967) the *Monopolistic Competition Revolution* was indeed quite a pathbreaking development in its time, as it replaced the Walrasian or Marshallian implicit "auctioneers" by explicit price setting agents internal to the economy, i.e. the firms. Since then an enormous amount of research has been devoted to this and related topics, but it seems fair to say that the domain of monopolistic competition has not reached the state of synthesis that the Walrasian system has reached [see notably Arrow and Debreu (1954), Debreu (1959), Arrow and Hahn (1971)]. The reason for this is that the theory of monopolistic competition (and more generally all theories which endogenize price making without an auctioneer) poses important and difficult conceptual problems. Our purpose in this chapter is to review a number of them. Of course, given the gigantic size of the literature on the subject, such a review can only be partial. As in the original monopolistic competition contribution we shall mostly concentrate on models with a generally large number of price setters, and indicate in the conclusion a number of alternative presentations.

The plan of the chapter is the following. Section 2 briefly reviews early developments in imperfect competition prior to Chamberlin. Section 3 introduces a basic model and studies problems of existence of an equilibrium. Section 4 discusses the issue of competitiveness of monopolistic competition. Section 5 introduces endogenous product differentiation. Section 6 considers general equilibrium representations of monopolistic competition. Section 7 presents macroeconomic applications. Subsections at the end of each section indicate a list of further reading.

[1] See also Robinson (1933) for an important contemporaneous contribution to imperfect competition.

2. History

We shall now start, just as Chamberlin did, by briefly reviewing a few models of imperfect competition with homogeneous goods, notably associated with the names of Cournot, Bertrand and Edgeworth, whose conceptual problems led Chamberlin to the idea of monopolistic competition.

2.1. The basic framework

We shall study here a market for a single homogeneous good, which may be served by several firms. We shall assume that the demand for this good is given by $q = D(p)$, and we shall denote the inverse demand curve as $p = F(q)$. In what follows we shall actually have to go beyond these basic data and make explicit where the demand curve comes from. We shall thus make a simple and usual assumption, i.e. that the consumer sector is made of a single "big" consumer with a utility function

$$U(q) = V(q) - pq$$

where output is implicitly paid in a numéraire commodity whose marginal utility is constant and normalized to one. Maximization of this with respect to q yields immediately

$$D(p) = V'^{-1}(p) , \qquad F(q) = V'(q) .$$

2.2. Cournot

Cournot (1838) first explored the case where the market is served by two firms with the same marginal cost c. These two firms are assumed to choose their quantities q_1 and q_2 independently. The resulting price is the one that "clears the market", i.e. $F(q_1 + q_2)$. The optimization program of firm 1 is thus

$$\text{maximize } F(q_1 + q_2)q_1 - cq_1 ,$$

yielding a best response function $q_1 = \psi_1(q_2)$. Symmetrically $q_2 = \psi_2(q_1)$. A Cournot equilibrium is characterized by quantities q_1, q_2 and a price p such that

$$q_1 = \psi_1(q_2) , \qquad q_2 = \psi_2(q_1) , \qquad p = F(q_1 + q_2) .$$

The Cournot price, though lower than the monopoly price, remains nevertheless strictly above the competitive price c.

2.3. *Bertrand*

Bertrand (1883) objected to Cournot's analysis on the basis that firms actually do set prices, and thus considered a model where prices are the strategic variables. In such a case a rule must be specified to allocate demand between the two competitors. Bertrand's rule is the following: if the prices are different, all demand will go to the lower price firm. If prices are equal, the demand is shared between the two firms, and we shall assume for simplicity that it is split half and half (the specific proportions actually do not matter here). As a result, the demand going to firm 1 is

$$D_1(p_1, p_2) = \begin{cases} D(p_1) & p_1 < p_2, \\ \frac{1}{2}D(p_1) & p_1 = p_2, \\ 0 & p_1 > p_2. \end{cases}$$

Clearly, as long as one price is above c, the other firm will have an incentive to undercut. As a result the unique possible equilibrium of this game is given by $p_1 = p_2 = c$ which is indeed the Nash equilibrium of this game. With prices as the strategic variables, two is enough for competition.

2.4. *Edgeworth*

Edgeworth (1897) in turn objected to Bertrand on the basis that one seldom sees productive processes with infinite potential supply, as costs must begin to rise at some point. Edgeworth thus considered the constant marginal cost case of Bertrand, but assumed there were fixed productive capacities k_1 and k_2.

The main change this brings to the previous analysis is that the demand to the higher price firm is no longer necessarily equal to zero. Indeed assume for example that $p_1 > p_2$, but $D_2(p_1, p_2) = D(p_2) > k_2$. We see that firm 2 cannot serve all demand addressed to it, and thus part of this demand will "come back" to firm 1. To see exactly to what extent, we must go back to the utility maximization program of our single consumer. With two prices p_1 and p_2 this program will be

$$\text{maximize } V(q_1 + q_2) - p_1 q_1 - p_2 q_2 .$$

If $p_1 > p_2$ and $D(p_2) > k_2$, then the consumer is rationed at price p_2 and buys exactly k_2 from firm 2. The demand to firm 1 is the solution in q_1 of the above program with $q_2 = k_2$. The first-order condition for an interior maximum is

$$V'(q_1 + k_2) = p_1 ,$$

yielding a demand equal to $D(p_1) - k_2$.[2] Summarizing, the actual demand to firm 1, its "contingent demand", which we shall denote as $\tilde{D}_1(p_1, p_2)$, is now

$$\tilde{D}_1(p_1, p_2) = \begin{cases} D(p_1) & p_1 < p_2 , \\ \max[\frac{1}{2}D(p_1), D(p_1) - k_2] & p_1 = p_2 , \\ \max[0, D(p_1) - k_2] & p_1 > p_2 . \end{cases}$$

A resulting profit function is shown in Figure 37.1 for $p_2 > c$ and $k_2 < D(p_2)$. We see that the undercutting argument which underlies Bertrand's result no longer works. In particular c cannot be an equilibrium in prices as the firms will always have an interest to jump to a higher price. One can easily check that

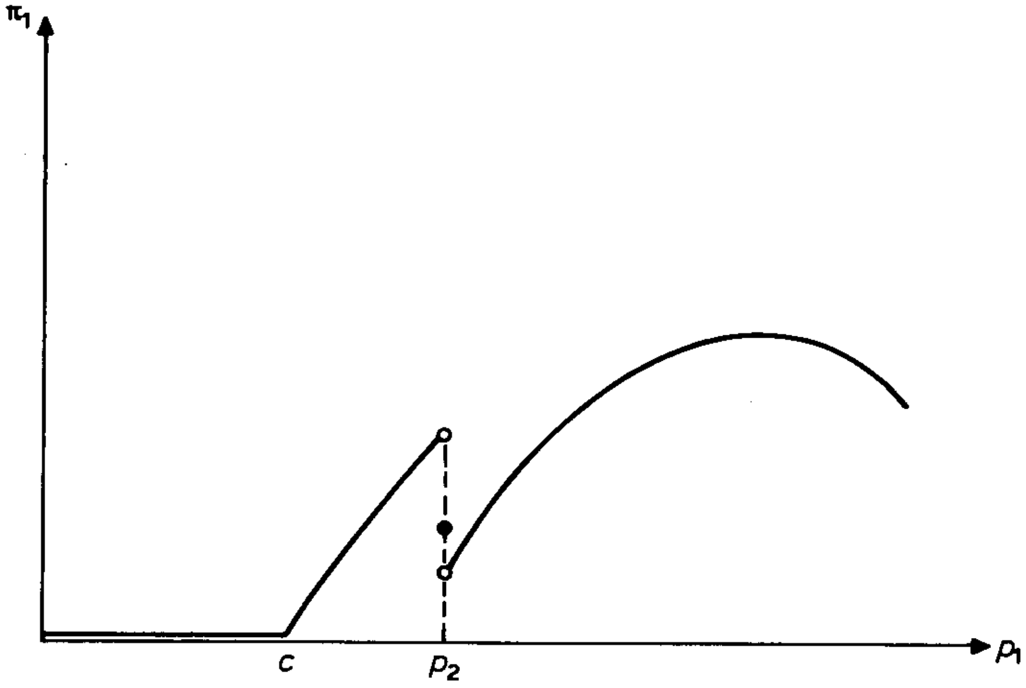

Figure 37.1

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[2] Note that the extreme simplicity of this demand comes from the fact that there is a single consumer and numéraire has constant marginal utility. For a thorough treatment of the general case, see Dixon (1987b).

there is no equilibrium in pure strategies with $p_1 = p_2 = c$ unless $\min\{k_1, k_2\} \geqslant D(c)$.

The above non-existence result is actually much more general than the particular example given by Edgeworth. It is indeed easy to see that for increasing marginal cost functions there is no Nash equilibrium in pure strategies for the price game [Shubik (1959), Dixon (1987a)].

3. A basic model and existence problems

Chamberlin comes thus at a point where the theory of endogenous price making by firms is somehow in a dead end because of Edgeworth's non-existence result. Chamberlin's way out will be to consider differentiated products, which will in particular eliminate the discontinuities in the demand curves associated with perfect substitutability. We should note that this idea of product differentiation had already been used in the pioneering contribution by Hotelling (1929) on spatial competition.[3]

3.1. A basic Chamberlinian model

Chamberlin thus considers n firms indexed by $j = 1, \ldots, n$ each producing a different good, also indexed by j. In order to reflect the fact that these products are imperfect substitutes, we shall assume that the utility functions of the agents in the consumer sector are strictly quasi-concave. To make exposition as simple as possible, let us again assume that this sector consists of a single "big" consumer with a utility function

$$U(q_1, \ldots, q_n, x) = U(q, x)$$

where x (a scalar) is a numéraire good representing somehow "the rest of the economy" and q is the vector of the n differentiated goods. The demands for goods $j = 1, \ldots, n$ will be simply given by the solution q of the following program:

$$\text{maximize } U(q, x) \text{ s.t. } pq + x = R$$

where R is the numéraire income of the consumer, assumed given in this partial equilibrium framework. We shall denote the solutions as

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[3] For a modern restatement of Hotelling's model and the corresponding existence problems, see notably D'Aspremont, Gabszewicz and Thisse (1979).

$$q_j = D_j(p_1, \ldots, p_n) = D_j(p_j, p_{-j})$$

where p_{-j} is the vector of all prices but p_j. We can now define an equilibrium with monopolistic competition [cf. for example Friedman (1982)].

Definition 1. An equilibrium with monopolistic competition consists of prices p_j^*, $j = 1, \ldots, n$ such that

$$p_j^* \text{ maximizes } p_j D_j(p_j, p_{-j}^*) - C_j[D_j(p_j, p_{-j}^*)] \quad \forall j .$$

We can easily relate this equilibrium to Chamberlin's traditional "short-run" equilibrium picture (Figure 37.2). Assume identical cost curves C_j and symmetrical demand curves D_j. The average revenue curve has for its equation

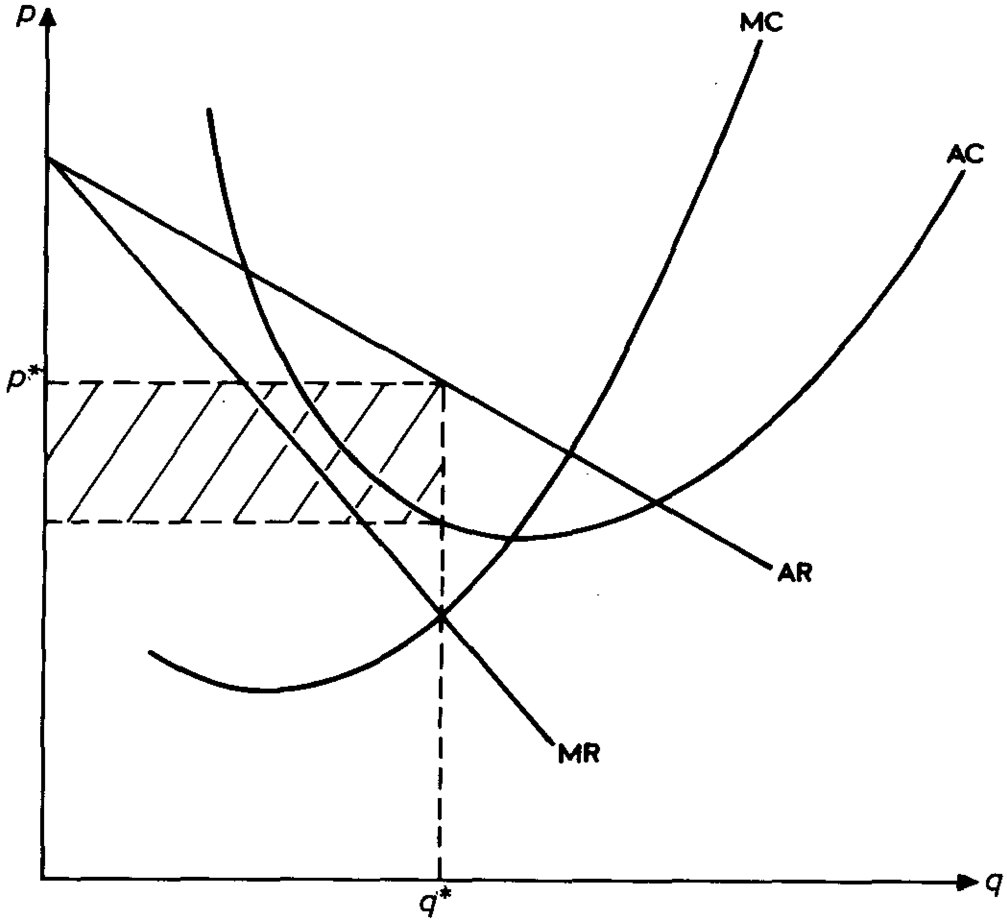

Figure 37.2

$$q = D_j(p, \bar{p}_{-j}),$$

i.e. it is the demand forthcoming to a representative firm, assuming all other firms' prices are held constant and equal to $\bar{p}$ (this curve depends thus on the value of $\bar{p}$). The short-run equilibrium is characterized by the equality of marginal cost and marginal revenue, with $\bar{p} = p^*$. Figure 37.2 displays a situation where firms can still earn a profit, represented by the shaded area. This corresponds to the equilibrium of Definition 1.

3.2. *A first existence problem*

As we shall see below, the problem of the existence of a monopolistic competition equilibrium will be a recurrent theme in this chapter. Taking the simple Definition 1, an equilibrium will be a fixed point of the mapping

$$p_j \rightarrow \arg\max \pi_j(p_j, p_{-j}), \quad j = 1, \ldots, n$$

where $\pi_j(p_j, p_{-j}) = p_j D_j(p_j, p_{-j}) - C_j[D_j(p_j, p_{-j})]$.

Whether the demand function D_j comes from one or many consumers, nothing in the traditional assumptions on utilities ensures that this mapping will have the required properties to have a fixed point. So it has been customary in the field to directly assume boundedness, convexity and upper-hemicontinuity of the above mapping (or to make assumptions trivially implying them; quasi-concavity in p_j of the profit functions π_j is a usual favorite) so that Kakutani's fixed point theorem can be applied.

It must be noted, however, that recently a few authors have sought not to use these assumptions directly. In particular Caplin and Nalebuff (1989), E. Dierker (1988), H. Dierker (1989) derive the quasi-concavity of the profit function from well-specified hypotheses on the distribution of consumers' characteristics.

3.3. *The Edgeworth problem*

A question we may now ask is whether product differentiation actually solves the Edgeworth non-existence problem, which was one of the main motivations for which Chamberlin (and others such as Hotelling) studied differentiated products. To make the issue particularly clear, we shall assume that the function $\pi_j(p_j, p_{-j})$ is strictly quasi-concave in p_j (Figure 37.3). We shall now show that, in spite of this, the Edgeworth non-existence problem may still arise because the function $\pi_j(p_j, p_{-j})$ is not the "true" profit function.

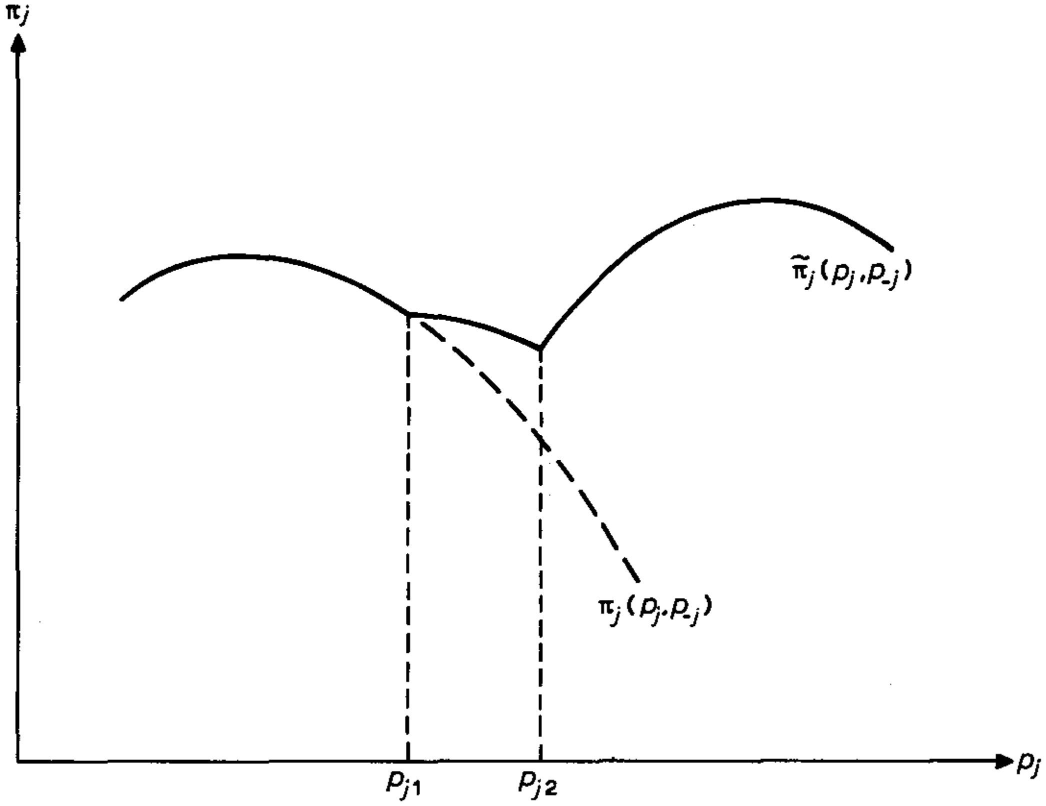

Figure 37.3

Indeed this profit function is based on the Walrasian demand $D_j(p_j, p_{-j})$, itself derived from the assumption that each firm will serve any demand at any price. Edgeworth pointed out that this could not possibly be true with fixed capacities, and we shall now see that, even without fixed capacities, this may also be inconsistent with profit maximization. Indeed let us consider some starting point (p_j, p_{-j}) and imagine that firm j considers raising its price p_j. If goods are gross substitutes (which we shall assume in all that follows), demand will be increasing for the competing products. However, it is clear that none of the competing firms, $i \neq j$, if they are true profit maximizers, will serve more than their profitable capacity $k_i(p_i) = C_i'^{-1}(p_i)$, so that the demand actually forthcoming to j, its "contingent demand", is solution of the program in q_j:

$$\textit{maximize } U(q, x) \text{ s.t.}$$
$$\begin{cases} pq + x = R\,, \\ q_i \leq k_i\,, \quad i \neq j\,, \end{cases}$$

which, since each k_i is a function of the corresponding p_i, yields a function

$\tilde{D}_j(p_j, p_{-j})$ which notably differs from the Chamberlinian one because of the quantity constraints $k_i(p_i)$. In particular, each time a competitor hits his capacity limit (which occurs at prices p_{j1}, p_{j2} in Figure 37.3), the function $\tilde{D}_j$ has a kink, becoming less elastic as more substitutes are rationed to the consumer. Consequently the "true" profit function $\tilde{\pi}_j$, given by

$$\tilde{\pi}_j(p_j, p_{-j}) = p_j\tilde{D}_j(p_j, p_{-j}) - C_j[\tilde{D}_j(p_j, p_{-j})] ,$$

is also kinked as in Figure 37.3. As a result $\tilde{\pi}_j$ need not be quasi-concave and existence may be jeopardized [Shapley and Shubik (1969), Benassy (1986b, 1989a)].

Of course, investigating existence in a general "Bertrand–Edgeworth–Chamberlin" model such as we have just described would be exceedingly difficult, and the problem has been investigated in symmetric models with identical cost curves and symmetric utility functions. The reader can find in Shapley-Shubik (1969) and Benassy (1989a) characterizations of how existence of a pure strategies equilibrium depends on the relations between the number of competitors, the degree of substitutability among the goods and the level of excess profitable capacities. It is shown in particular in Benassy (1989a) that a sufficient condition for the traditional Chamberlin equilibrium still to be a Nash equilibrium in this model is that

$$(n-1)(k^* - q^*) \geqslant q^* , \tag{1}$$

i.e. that excess productive capacities of the competitors be greater than each firm's production at the Chamberlin equilibrium q^*, a quite intuitive condition. Conversely if q^* is sufficiently greater than excess capacities, the equilibrium in pure strategies can be destroyed.

All this shows quite clearly that, contrary to a traditional belief, consideration of differentiated commodities only partially solves the existence problem which Edgeworth posed in the case of perfect substitutes.

3.4. *The Chamberlinian model with entry*

As we indicated, the equilibrium studied in the two preceding subsections is a short-run one, which generates profits for all firms, as shown in Figure 37.2. Now Chamberlin assumes that such an equilibrium with positive profits cannot last, as the mere existence of these profits will lead to entry of new firms. As a result, the demand curve (and the associated marginal revenue curve) will

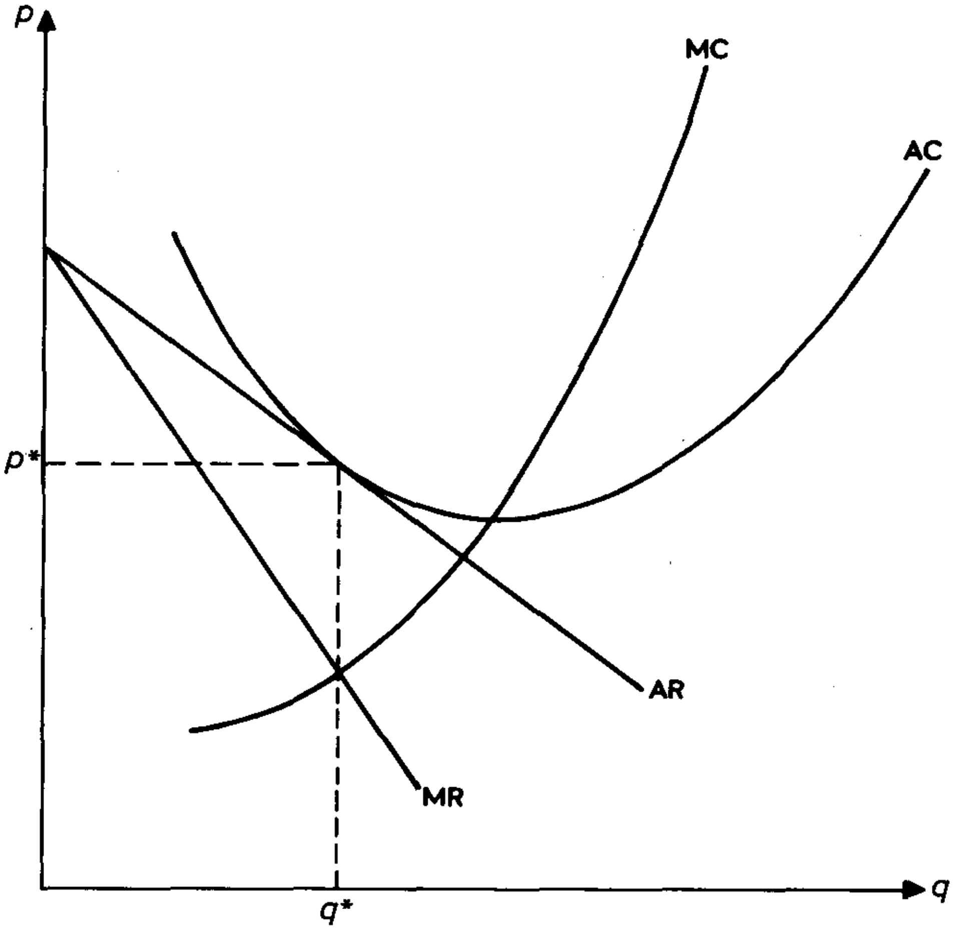

Figure 37.4

move to the southwest until one reaches the famous tangency condition (Figure 37.4) where all profits have been wiped out by entry.

Now we should note that, as compared to the equilibrium without entry (Definition 1 and Figure 37.2), there is a very serious conceptual problem associated with the potential definition of the equilibrium with entry implicit in Figure 37.4. Indeed we used in Definition 1 the "traditional" formalization of an equilibrium where the set of goods is given a priori. Entry in this framework means that we are adding new goods to the list of goods, and thus changing the space of goods in which we are working. Even if adding new firms does not pose much problem if they are assumed to have identical technologies, it is far from clear how preferences in the "old" and "new" space will relate to each other, and in particular how to derive them from underlying characteristics of potential goods. We shall see in Section 5 that there are many different approaches to this problem, but first we shall consider an economy with a given set of goods and tackle an important problem, that of the "competitiveness" of a monopolistic competition equilibrium.

3.5. Further reading

Existence of a pure strategies price equilibrium is further investigated in Vives (1990). In the case where a pure strategies Nash equilibrium does not exist, one may look for mixed strategies equilibria [see Glicksberg (1952) for continuous payoff functions and Dasgupta and Maskin (1986) for discontinuous ones].

4. How competitive is monopolistic competition?

In this section we study an important conceptual and practical question, that is, how close to perfect competition is a monopolistic competition equilibrium. In order to be precise, we have to choose an index of competitiveness. We shall take Lerner's "degree of monopoly":

$$L_j = \frac{p_j^* - C'(q_j^*)}{p_j^*} \,.$$

The closer to zero this index is, the more "competitive" the market. Now at least two factors are often cited in the literature as conducive to a close to competitive outcome. The first is often referred to as "market size"; competition will obtain if each competitor is small as compared to the market he operates in. A second factor is substitutability; a market will be competitive if competitors produce goods which are very close substitutes to the goods you produce. We shall now study how various concepts of imperfect competition allow us to relate competitiveness to these two factors.

4.1. The Cournot equilibrium and market size

We shall now see that the Cournotian model quite naturally leads to market size as a fundamental determinant of competitiveness. Assume thus there are n firms producing perfectly substitutable goods with cost functions $C_j(q_j)$, $j = 1, \ldots, n$. Call Q total production and $F(Q)$ the inverse demand curve.

Definition 2. A Cournot equilibrium is defined by a set of quantities q_j^*, $j = 1, \ldots, n$ and a price p^* such that:

(a) q_j^* maximizes $F(q_j + \Sigma_{i \neq j} q_i^*)q_j - C_j(q_j)$, $\forall j$,

(b) $p^* = F(Q^*) = F(\Sigma_j q_j^*)$.

Now the first-order condition of the maximization program giving q_j^* yields immediately

$$F(Q) + q_j \cdot F'(Q) - C_j'(q_j) = 0 .$$

Thus the Lerner index is easily computed as

$$\frac{F(Q) - C_j'(q_j)}{F(Q)} = -\frac{q_j}{Q} \cdot \frac{QF'(Q)}{F(Q)} = \frac{q_j}{Q} \cdot \phi$$

where ϕ is the absolute value of the elasticity of the inverse demand curve.[4] We thus see that, other things equal, the Lerner index is proportional to q_j/Q, i.e. the size of firm j's production as compared to the total production of the good, which will be equal to $1/n$ if the model is symmetric.

We should point out that the relation between "market size" and competitiveness has been quite refined beyond the above computations based on the number of competitors. In a series of contributions [Novshek and Sonnenschein (1978), Novshek (1980) and several others], competitiveness is related to the ratio of optimum productive size to demand at minimal cost (there are thus increasing returns). Useful surveys of this important line of research can be found in Fraysse (1986), Mas-Colell (1982) and Novshek and Sonnenschein (1986, 1987). We shall see other generalizations of the above idea in Section 6.3.

4.2. *The traditional Chamberlinian model and substitutability*

Let us now consider the Chamberlinian model presented in Definition 1, where the products $j = 1, \ldots, n$ are imperfect substitutes. Recall that the program leading to p_j is written

$$\text{maximize } p_j D_j(p_j, p_{-j}) - C_j[D_j(p_j, p_{-j})] .$$

The first-order conditions lead to

$$L_j = \frac{p_j - C_j'(q_j)}{p_j} = \frac{1}{\eta_j}$$

where $\eta_j = -(p_j/D_j)\, \partial D_j/\partial p_j$. We want now to relate this own-price elasticity of the demand curve D_j to more basic parameters. Recall that the demand D_j is obtained by maximization of the utility $U(q, x)$ under the budget constraint

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[4] Note that, though the goods $j = 1, \ldots, n$ considered are perfect substitutes, ϕ represents somehow an index of substitutability with the other goods in the economy, as we shall see in the next subsection.

$$pq + x = R\,.$$

Denote by σ_{ij} the Allen–Hicks elasticity of substitution between goods i and j [Allen and Hicks (1934), Allen (1938)]. This is related to the term s_{ij} of the Slutsky matrix by

$$\sigma_{ij} = \frac{R}{q_i q_j}\, s_{ij}\,.$$

Using the Slutsky relation it is easy to compute

$$\frac{\partial \log D_i}{\partial \log p_j} = \frac{p_j q_j}{R}\,(\sigma_{ij} - e_{iR})$$

where e_{iR} is the income elasticity of D_i with respect to R. Now using this formula and differentiating the budget constraint with respect to p_j we obtain

$$\eta_j = 1 + \sum_{i \neq j} \frac{p_i q_i}{R}\,(\sigma_{ij} - e_{iR}) + \frac{x}{R}\,(\sigma_{xj} - e_{xR})\,.$$

We see that η_j is equal to 1 plus a weighted sum of the elasticities of substitution of good j with goods $i \neq j$ and the numéraire x.[5] What this expression shows us is that in this Chamberlinian model the number of competitors does not really matter in determining competitiveness. What matters is the degree of substitutability among the goods, as described by the elasticities of substitution. In particular considering the limit case where two of the goods are perfect substitutes, we obtain the Bertrand result that "two is enough for competition". We have thus now quite clearly obtained substitutability as a factor of competitiveness, but, compared to the Cournot model, this is at the price of the disappearance of market size as a factor of competitiveness, which is somewhat unfortunate. We shall now see in the next two subsections that Edgeworth's qualification of the Bertrand and Chamberlin models does yield a significant role to market size in a market with explicit price makers.

4.3. Bertrand–Edgeworth and market size

Let us now consider the Bertrand–Edgeworth model of Section 2.4, but assume this time that there are n firms, each with marginal cost c and capacity

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[5] With more than one consumer we would obtain a weighted sum (this time across consumers) of similar expressions for each consumer.

k_j. Call K total capacity. We shall now see that in this Bertrand–Edgeworth model competitiveness does relate to size.

As a simple example, let us first consider the case where $D(c) < K$ (Figure 37.5). The reader can easily check that the price $p = c$ will be a Nash equilibrium for the Bertrand–Edgeworth game provided that

$$\max_j k_j \leqslant K - D(c)$$

i.e. there must be excess capacity in the market as a whole, and this excess capacity must be greater than the maximum capacity of every single competitor. The intuitive reason behind this condition is that in such a case all demand lost by a high price firm can actually be served by the other competitors. If all competitors have the same capacity, this can be rewritten as

$$\frac{K - D(c)}{K} \geqslant \frac{1}{n}$$

i.e. the relative excess capacity must be greater than $1/n$, which naturally relates competitiveness to market size.

If we now move to the cases where a pure strategies equilibrium does not exist, a number of studies have shown that a different form of Nash equilibrium could exist, and would somehow "converge" towards c as n became large.

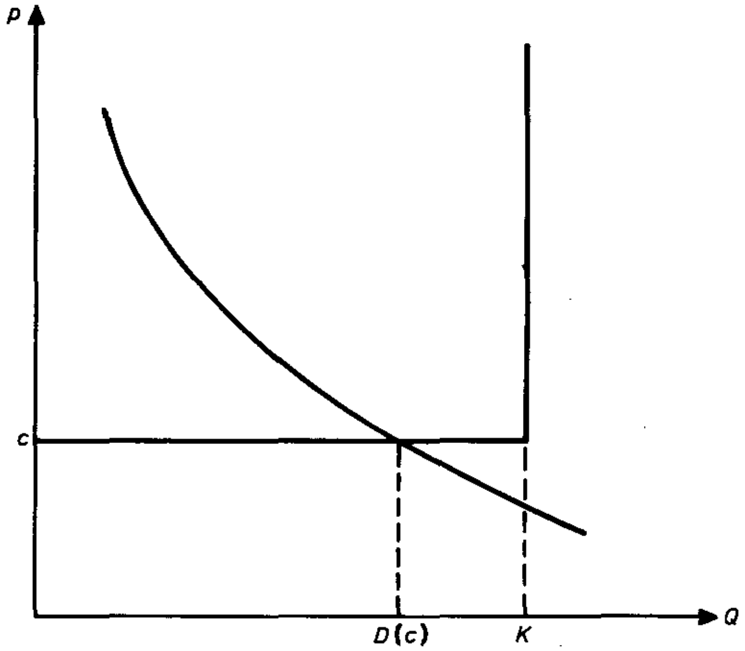

Figure 37.5

Shubik (1959), Allen and Hellwig (1986a,b) and Vives (1986) have shown that mixed strategies equilibria would exist in the Bertrand–Edgeworth game, and converge in probability towards the competitive price. Similar results are obtained by Dixon (1987a) for a concept of approximate Nash equilibrium. Introducing quantity constraints in the Bertrand model thus allows us to re-introduce market size as a main determinant of competitiveness, both in pure and mixed strategies senses.

4.4. *The Chamberlinian model revisited*

In light of the above Bertrand–Edgeworth formalization, we now see that there was not much chance for market size to play a role in the traditional Chamberlinian model, as it is implicitly assumed in the construction of the demand curve that each firm can (and will) serve any amount of demand forthcoming at any price. Under this assumption any small firm is assumed to be potentially able to serve the whole market demand, and under such circumstances it must come as no surprise that market size does not matter in the competitiveness of a traditional Chamberlinian equilibrium.

We saw however in Section 3.3, that the assumption that any firm would serve any demand forthcoming at any price was not consistent with profit maximization, and should be replaced by the assumption that each firm would not serve more than its profitable capacity. Under such circumstances market size naturally re-appears in the corresponding model. We saw indeed [equation (1)] that a sufficient condition for the Chamberlin equilibrium still to be a Nash equilibrium in this model is that

$$(n-1)(k^* - q^*) \geqslant q^* \tag{1}$$

where profitable capacity k^* is defined by

$$C'(k^*) = p^* . \tag{2}$$

To get an intuitive grasp of how (1) relates competitiveness to size, let us consider an example and assume isoelastic demand curves (this is actually derived from a specific utility function in Section 5.1)

$$q_j = \lambda_j p_j^{-\varepsilon} , \quad \varepsilon > 1 , \tag{3}$$

as well as cost functions of the form

$$C_j(q_j) = cq_j^{\beta} + f , \quad \beta \geqslant 1 , \tag{4}$$

i.e. constant ($\beta = 1$) or increasing ($\beta > 1$) marginal costs. Under (3), the traditional equality between marginal revenue and marginal cost is written

$$C'(q^*) = p^*\left(1 - \frac{1}{\varepsilon}\right), \tag{5}$$

which allows us to compute the Lerner index

$$L^* = \frac{p^* - C'(q^*)}{p^*} = \frac{1}{\varepsilon}. \tag{6}$$

While using (2), (4) and (5), equation (1) is rewritten

$$\left(\frac{\varepsilon}{\varepsilon - 1}\right)^{1/(\beta - 1)} - 1 \geqslant \frac{1}{n - 1}$$

which, using simple manipulations, yields the slightly stronger condition

$$n - 1 \geqslant (\beta - 1)\varepsilon. \tag{7}$$

Though (7) is only a sufficient condition for the existence of a pure strategies equilibrium, its discussion is quite enlightening.

We first see that for perfect substitutes (ε infinite) and increasing marginal costs ($\beta > 1$), condition (7) is never satisfied, which corresponds to the Edgeworth non-existence problem. Secondly for $\beta = 1$ (constant marginal costs), condition (7) imposes no constraint, and competitiveness only requires high ε, corresponding to high substitutability. In this case "two is enough for competition".

If, however, marginal costs are increasing ($\beta > 1$), we see immediately that condition (7) will be satisfied for large ε (i.e. by (6) for near competitive outcomes) only if n itself is suitably large. The lesson from these simple calculations is clear: unless marginal costs are constant throughout (which is quite unrealistic), competitiveness in a price setting game results from two factors: (a) the existence of close substitutes, so that the Walrasian demand has a high elasticity; (b) a large market size, which in the above framework yields sufficient unused capacities for competing products.

We should finally point out that we have only considered here pure strategies equilibria. Intuition suggests that a full characterization of equilibria, for example in terms of mixed strategies Nash equilibria, would make both the degree of substitutability and market size appear as determinants of competitiveness, but such characterization unfortunately does not yet exist.

4.5. Further reading

As we have seen here, in the traditional Chamberlinian model competitiveness is clearly related to substitutability between competing goods, but not to market size. In order to validate the important "Cournotian" insight that market size also plays an important role, we introduced (rationally perceived) quantity constraints, as in Edgeworth. But a full characterization of the resulting model still remains to be done in the imperfect substitutes case. Another interesting link between the Cournotian and Bertrand–Edgeworth lines has been studied by Kreps and Scheinkman (1983) who showed that a first round capacity competition followed by a second round Bertrand–Edgeworth price competition could lead to a Cournotian-type equilibrium. Though the exact equivalence has been shown to be fragile (Davidson and Deneckere 1986), this may be a line worth pursuing.

In a different vein, direct comparison of the competitiveness of price versus quantity competition for a given demand system is carried out in Vives (1985).

Finally an alternative way of making market size appear as a determinant of competitiveness, while staying within the "traditional" Chamberlinian approach, is to assume that goods must become very close substitutes as their number becomes large, based on the idea that there is limited diversity of potential goods, an insight found in Mas-Colell (1975), Hart (1979) and Jones (1987). In order to better study this issue we must now move to models where the number of competitors and the nature of products is endogenous.

5. Endogenous product differentiation

We have so far considered economies with a given number of products and operating firms. Consideration of the entry phenomenon in the Chamberlinian model now leads us to consider models where the set of products and of firms is itself endogenous, and therefore to examine in more detail the issue of product differentiation.

5.1. The modelling of product differentiation and entry: a first approach

The method usually employed to formalize product differentiation is to postulate a set of "potential" goods, which is a priori quite bigger than the set of goods actually produced, and to specify both the preferences of consumers and the productive possibilities of potential firms in terms of these potential goods.

A first and popular method is that of the "representative consumer" approach [see for example Dixit and Stiglitz (1977), Spence (1976)]. A typical model assumes that there is a set of countably infinite potential goods. Each is produced by a different firm so that the (endogenous) number of firms will be equal to that of the differentiated goods. The consumption sector is assumed to be represented by a "big" consumer with income R and a utility function

$$U(q_1, \ldots, q_n, x) \tag{8}$$

where this time the utility function U must be defined for any value of n. Quite often a particular parametrization is chosen, for example involving a subutility index for the differentiated goods, such as a CES one,

$$U(q_1, \ldots, q_n, x) = V\left[\left(\sum_{j=1}^{n} q_j^{\theta}\right)^{1/\theta}, x\right] \tag{9}$$

where $0 < \theta < 1$ and V is homothetic. Note that such a function displays "preference for diversity", as the consumer will always want to consume some amount of each of the n goods available. Maximization of a function such as (9), subject to the budget constraint

$$\sum_{j=1}^{n} p_j q_j + x = R ,$$

yields for large n approximately isoelastic demand curves for each product j of the form

$$q_j = \lambda_j p_j^{-\varepsilon} \tag{10}$$

with $\varepsilon = 1/(1-\theta)$. To solve such a model fully, let us assume that each potential firm operates with constant marginal cost c and a fixed cost f, and further specify the utility function (9) as

$$U(q_1, \ldots, q_n, x) = \left(\sum_{j=1}^{n} q_j^{\theta}\right)^{\alpha/\theta} x^{1-\alpha} . \tag{11}$$

First take the number n as given (short-run equilibrium). Each firm maximizes profit $(p_j - c)q_j - f$ subject to the demand constraint (10), which yields immediately

$$\bar{p} = \frac{c}{1-(1/\varepsilon)} = \frac{c}{\theta} . \tag{12}$$

Secondly maximization of utility function (11), assuming all prices p_j equal to p, yields

$$q_j = \frac{\alpha R}{np} \tag{13}$$

so we obtain with the help of (12),

$$\bar{q} = \frac{\theta \alpha R}{nc} . \tag{14}$$

Now $\bar{p}$ and $\bar{q}$ represent short-run equilibrium values relative to a given n. The number of products actually produced will be given by the zero profit condition

$$(\bar{p} - c)\bar{q} - f = 0 ,$$

yielding the equilibrium number of firms

$$n^* = \frac{(1-\theta)\alpha R}{f} \tag{15}$$

and, with the help of (12) and (14) the equilibrium price and quantity

$$p^* = \frac{c}{\theta} , \qquad q^* = \frac{\theta f}{(1-\theta)c} . \tag{16}$$

We may note that this model displays the particularity that, even if the number of firms goes to infinity, which will occur for example if f/R goes to zero, the price will nevertheless remain bounded away from the competitive price c, due to the CES form of the subutility function in (9). A similar result can actually be obtained with a multitude of consumers consuming each a finite number of goods [Hart (1985b)].[6]

As we have just seen, the representative consumer approach to product differentiation allows us to characterize quite easily a situation of Chamberlinian equilibrium with an endogenous number of goods and firms. It poses however serious problems of interpretation, as it is notably quite unclear from which underlying characteristics of the potential goods particular families of utility functions as in (8), (9) or (11) come. For this reason a substantial body of literature has developed to examine this issue.

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[6] Other symmetric Chamberlinian type models with a multitude of consumers are built in Perloff and Salop (1985) and Sattinger (1984).

5.2. *Product differentiation*: *a general view*

In this more general view of product differentiation, found for example in Mas-Colell (1975), it is assumed that a good is fully described by a set of characteristics, which may include the full physical description of the good, location and time of availability, etc. Each potential good is described by a point in a characteristics set K. In that framework a consumption plan or a production plan are represented by a measure on K. Consumption sets or production sets are the sets of all feasible such measures. Preference relations can be defined on these measures. An assumption of continuity of preferences allows us to define a notion of "closeness" or "substitutability". Two goods will be highly substitutable if they are topologically close (Mas-Colell, 1975).

Of course it is difficult to obtain general results in monopolistic competition equilibria using such a general characterization. Research has thus proceeded along several lines, each using a particular, and often more intuitive, specification of characteristics and preferences. There are a few well-known examples.

(a) The original "characteristics" approach notably pioneered by Lancaster (1966, 1975, 1979) assumes that what the consumers are interested in is a set of l characteristics. A good "k" is described by the vector of quantities of each characteristic $k_1, \ldots, k_l$ which it embodies. The set K is the set of vectors k corresponding to technologically feasible goods.[7] If a consumer consumes a "distribution" $\mu(K)$ on K, his utility will be given by $U(q, x)$, where x is a numéraire good and $q \in R^l$ is the sum of characteristics obtained via the distribution μ, i.e.

$$q = \int_K k \mathrm{d}\mu(k) .$$

(b) In models of vertical differentiation, the characteristic is for example a "quality" variable, s. The name "vertical differentiation" comes from the fact that at equal prices, all consumers will rank goods in the order of descending s. For example Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982, 1983), use utility functions of the following form (assuming one unit of only one quality s is consumed)

$$U = sx = s(R - p(s))$$

where R is numéraire income, $p(s)$ the price of quality s and x the numéraire left. An interesting feature of these models of pure vertical differentiation is the "finiteness property" according to which there is a maximum number of

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[7] K will be defined in an $(l-1)$-dimensional subspace to avoid colinearities.

firms which can co-exist with a positive market share at a free entry equilibrium [see Gabszewicz and Thisse (1980), Shaked and Sutton (1983, 1987)].

(c) Conversely, in models of horizontal differentiation, no good is everybody's first choice, and which product will be chosen at equal prices depends on the consumer. The consumer sector is generally represented by a distribution of consumers, each with a different ranking of the goods. A typical model of horizontal differentiation is the spatial competition model, which started with Hotelling (1929) and of which we shall now give an example.

5.3. Spatial competition

We shall consider in this section a model of horizontal differentiation which comes fairly close to the ideas of monopolistic competition and will allow us to show on a precise example how to model both the price decisions of firms and the endogenous determination of the range of products offered to the consumers. This is the so-called spatial model of monopolistic competition which depicts spatial competition "around a circle" [see Salop (1979)]. The consumer sector is depicted by a circular market of length L, along which consumers are uniformly distributed with a density Δ. These consumers must travel along the circle to purchase output from a firm, and we shall assume that in so doing a consumer incurs transportation costs amounting to τd^2, where d is the distance travelled between the firm and the consumer. Each consumer is assumed to demand inelastically one unit of output.

On the production side, we assume there is a very large number of "potential" firms. Each firm which enters the market bears a fixed cost f, and thereafter has constant marginal costs c. Products are differentiated from the point of view of the consumers, since they must bear the transportation costs. Firms must decide sequentially (i) whether or nor to enter, (ii) which "good" to produce (i.e. where to locate in the circle) and (iii) which price to set.

Let us first consider the post-entry stage and suppose that n firms have entered. It has been shown by Economides (1989) that with quadratic transportation costs, the firms will seek maximal differentiation from each other in the location game, and thus that they establish at equidistant locations so that each firm has two nearby competitors at distance L/n. We shall look for a symmetric equilibrium, so consider a firm j surrounded by other firms who have all set prices $\bar{p}$. Then a consumer situated at distance $\zeta \in [0, L/n]$ from firm j will be indifferent between j or one of his neighbors if

$$p_j + \tau\zeta^2 = \bar{p} + \tau\left(\frac{L}{n} - \zeta\right)^2 ,$$

which easily solves in ζ and yields demand to firm j,

$$D_j = 2\zeta\Delta = \left(\frac{L}{n} - \frac{n}{\tau L}(p_j - \bar{p})\right)\Delta .$$

Firm j maximizes $(p_j - c)D_j$, which yields

$$\frac{L}{n} = \frac{n}{\tau L}(p_j - \bar{p}) + \frac{n}{\tau L}(p_j - c)$$

and allows us to compute prices and profits at a symmetric equilibrium,

$$p_j = \bar{p} = c + \frac{\tau L^2}{n^2}, \qquad \bar{\pi} = \frac{\Delta\tau L^3}{n^3} - f .$$

Now, in the "first" stage of the game, entry will proceed until this profit has been drawn to zero, which immediately gives the equilibrium values

$$n^* = L\left(\frac{\Delta\tau}{f}\right)^{1/3}, \qquad p^* = c + \tau\left(\frac{f}{\Delta\tau}\right)^{2/3} . \tag{17}$$

This model of spatial monopolistic competition will allow us to study in a simple manner a number of important issues such as competitiveness, efficiency and the zero profits assumption.

5.4. *Competitiveness*

Looking at equations (17), we first see that we may have a large variety of equilibrium situations, as far as competitiveness is concerned:

(a) a very competitive situation with a small number of firms if τ is low, which corresponds somehow to very high substitutability (note that the Edgeworth problem does not arise here since marginal costs are constant and there is no capacity limit),

(b) a very competitive situation with a large number of firms, which will occur if f is low or Δ is high;

(c) but we may also have a non-competitive situation with a large number of firms if L is high.

We may first observe that, in accordance with our discussion of Section 4.2, competitive situations occur here when the goods of two competing firms have become very substitutable. But this situation itself may come from two different causes. In case (a), all goods in the characteristics space (the circle) are highly substitutable because of low transport costs. In that case, two or a small number of firms is enough for competitiveness. In case (b), the market can support a large number of competitors which somehow "crowd in" the restricted characteristics space, so that each firm has two nearby competitors which produce goods which are very substitutable to his. We should note that this last insight has been studied in more generality by Jones (1987) who showed that in a two stage game where firms choose first the type of good they will produce, and then prices, a large number of operating firms (due to small fixed costs) will lead to a near competitive outcome if the set of possible products is compact.

A second remark, inspired by the comparison of cases (b) and (c) is that whether or not large numbers of competitors lead to a competitive outcome depends very much on which underlying parameter (or combination or parameters) leads in the first place to a large number of operating firms. Notably, a quick look at formulas (17) shows that one can easily construct examples where the number of firms tends to infinity while the price does not converge towards its competitive value. In particular, case (c) shows that convergence to competition can fail to obtain if the increase in numbers is due to a larger set of characteristics.

5.5. Efficiency

The model of spatial monopolistic competition also allows a simple investigation of the problem of the optimum number of firms (and thus of products). From the social point of view, it would be optimal to minimize the sum of transportation costs and fixed costs, i.e.

$$n \int_{-L/2n}^{L/2n} \Delta\tau\zeta^2 \, \mathrm{d}\zeta + nf = \frac{\Delta\tau L^3}{12n^2} + nf \, .$$

Minimization of that function yields the optimal number of firms n^{opt},

$$n^{\mathrm{opt}} = L\left(\frac{\Delta\tau}{6f}\right)^{1/3} < L\left(\frac{\Delta\tau}{f}\right)^{1/3} = n^* \, .$$

We thus see that at the monopolistically competitive equilibrium there will always be too many firms and products, i.e. there is excessive product diversity.

We can also compare the optimal number of firms and production level in the simple model of Section 5.1. With n firms, the remaining amount of numéraire, once production costs are covered, is

$$x = R - nf - c\sum_{j=1}^{n} q_j ,$$

so that the optimum will be obtained by unconstrained maximization of

$$\left(\sum_{j=1}^{n} q_j^{\theta}\right)^{\alpha/\theta}\left(R - nf - c\sum_{j=1}^{n} q_j\right)^{1-\alpha}$$

which yields

$$q^{\text{opt}} = \frac{\theta f}{(1-\theta)c} ,$$

$$n^{\text{opt}} = \frac{(1-\theta)\alpha R}{(\theta + \alpha - \alpha\theta)f} .$$

Comparison with the equilibrium values q^* and n^* [equations (15) and (16)] shows that

$$q^{\text{opt}} = q^* , \qquad n^{\text{opt}} > n^* ,$$

i.e. this time there is insufficient product diversity, even though the level of production is the correct one.

Of course both results are particular to the two specific models studied here. Dixit and Stiglitz (1977) and Spence (1976) have shown in the framework described in Section 5.1 that almost any configuration of n^{opt} and n^*, q^{opt} and q^*, could obtain by suitably choosing the utility function.

An interesting byproduct of the above computations is to show that the presence of "excess capacity" at the monopolistically competitive equilibrium is not per se a proof of inefficiency, as was believed for some time after Chamberlin, since the optimum also takes place here in the decreasing portion of the average cost curve.

5.6. *Zero profits*

The characteristics approach, and notably spatial competition theory, also allows us to re-examine one of the basic assumptions of traditional monopolistic competition, i.e. that free entry leads to zero profits. In particular a number of authors have studied sequential entry in location models, and shown that free entry was consistent with positive long-run profits if each producer had to commit irrevocably to a particular location (i.e. a product with given characteristics) when entering [Eaton and Lipsey (1978), Eaton and Wooders (1985), Hay (1976), Prescott and Visscher (1977)]. Indeed the zero profit condition actually comes from two distinct sources: (a) entry occurs if potential profits are non-negative; (b) after entry profits of all firms are the same. Clearly free entry corresponds to (a) only. Condition (b) is a consequence of particular formalizations. In particular, in the spatial monopolistic competition model, (b) comes from the fact that all firms costlessly and symmetrically relocate after they have all entered. If entry is sequential and relocation is costly or impossible, then pure profits will subsist in the long run even if entry and location decisions are fully rational.

The following simple example, inspired from one in the insightful survey by Gabszewicz and Thisse (1986) will illustrate the point. Consider the model of competition on the circle (Section 5.3), and assume $\Delta\tau = f$ and $L = 4$, so that in the equilibrium where all firms enter simultaneously, there would be exactly four firms at distance 1 from each other, charging the same price $c + \tau$ and making exactly zero profits (Figure 37.6a).

Imagine now that firms enter sequentially in the order $1, 2, 3, \ldots$. Firm 1 will locate anywhere on the circle. Clearly if firm 2 locates at distance $2 - \varepsilon$ from firm 1, only firm 3 will be able to enter and will choose the location at distance $1 + (\varepsilon/2)$ from firms 1 and 2 (Figure 37.6b). Firm 4 cannot enter in the other segment between firms 1 and 2, as it would not cover fixed costs. Thus at equilibrium there are only three firms. Taking an infinitesimal ε, we find after tedious calculations that

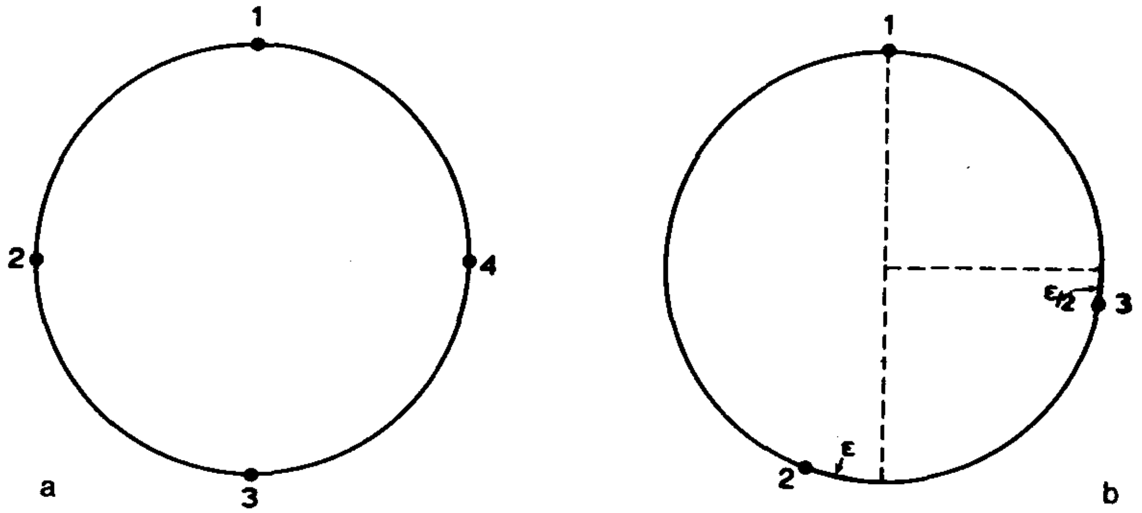

Figure 37.6

$$p_1 = p_2 = c + \frac{7\tau}{4}, \qquad p_3 = c + \frac{11\tau}{8},$$

$$\pi_1 = \pi_2 = \frac{83f}{64}, \qquad \pi_3 = \frac{57f}{64}.$$

In that case the persistence of positive profits in the long run is consistent with free entry.

5.7. Further reading

A problem which immediately strikes the reader of this domain is the large variety of different formalizations of product differentiation, a variety which the above compact presentation very much understates. For a more complete view, the reader may consult the surveys of Archibald, Eaton and Lipsey (1985), Encaoua (1990), Ireland (1987), as well as the survey on spatial competition in Gabszewicz and Thisse (1986).

Fortunately a number of authors have recently tried to draw bridges between various approaches. Anderson, De Palma and Thisse (1987) show that the representative consumer approach can be derived from the characteristics approach with an adequate distribution of characteristics. It turns out that with n products, the dimension of the characteristics space is $n-1$. Deneckere and Rothschild (1989) construct a synthetic model which admits as particular cases the Chamberlinian symmetric model and the model of spatial competition on the circle.

6. General equilibrium representations

All the previous developments have been cast in a fairly partial equilibrium framework in order to make the conceptual problems clearer. Already in 1940, Triffin had forcefully advocated a full general equilibrium approach. We shall thus develop in this section a number of general equilibrium representations of monopolistic competition. For that we shall revert to an a priori given set of goods, as really general concepts have been developed in that case only.

6.1. General framework

We shall consider here an economy with m households $i = 1, \dots, m$ and n firms $j = 1, \dots, n$. Goods exchanged in this economy are a numéraire good and non-monetary goods $h \in H = \{1, \dots, l\}$, with prices p_h.

Firm j has a production vector $y_j \in R^l$ which must belong to a production set $Y_j \subset R^l$ with $0 \in Y_j$.[8] We shall assume that the objective of the firm is to maximize profits $\pi_j = py_j$.

Household i has a vector of endowments of goods and numéraire $\omega_i \in R^l_+$ and $\bar{x}_i \geqslant 0$, and a utility function $U_i(w_i + z_i, x_i)$ where z_i is the l-dimensional vector of net trades in goods and x_i is the final holding of numéraire good. The household maximizes utility subject to his budget constraint, which reads

$$pz_i + x_i = \bar{x}_i + \sum_{j \in J} \theta_{ij} \pi_j$$

where θ_{ij} is household i's share of firm j's equity.

6.2. Subjective demand curves

We shall now describe a concept of general equilibrium with subjective demand curves, as developed in the pathbreaking work of Negishi (1961, 1972). It will be assumed that the price makers are the firms (the concept actually extends easily to price making households). Call H_j the set of goods whose prices are controlled by firm j. We have

$$H_j \cap H_{j'} = \{\emptyset\}, \quad j \neq j',$$

i.e. each good has its price set by at most one firm. The fundamental element in the decisions of the firm is the subjective demand curve which shows how much the firm expects to sell as a function of the price it sets. This expectation is subjective, hence the name of subjective demand curves.

Negishi actually uses a perceived inverse demand curve which shows at which price the firm expects to be able to sell an output as a function of the quantity put on the corresponding market. This inverse curve is denoted as

$$p_h = P_h(y_{jh}, \bar{p}, \bar{y}_{jh}), \quad h \in H_j$$

where $\bar{p}$ and $\bar{y}_{jh}$ are the observed price vector and production of good h (we shall see below in the definition of equilibrium that these will be those actually observed at equilibrium). This perceived demand curve satisfies a natural consistency condition,

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[8] We may note that the numéraire does not enter the production sets. This assumption is solely made to simplify notation and to facilitate transition to the next section where the numéraire good is explicitly assumed to be fiat money.

$$\bar{p}_h = P_h(\bar{y}_{jh}, \bar{p}, \bar{y}_{jh}), \quad \forall h \in H_j,$$

i.e. the curve "goes through" the observed point [Bushaw and Clower (1957)].

Definition 3. An equilibrium with subjective demand curves is defined as a set of p^*, z_i^*, $i = 1, \ldots, m$, y_j^*, $j = 1, \ldots, n$ such that

(a) z_i^* maximizes $U_i(\omega_i + z_i, x_i)$ s.t. $p^* z_i + x_i = \bar{x}_i + \Sigma_j \theta_{ij} p^* y_j^*$,

(b) y_j^* maximizes $\Sigma_{h \notin H_j} p_h^* y_{jh} + \Sigma_{h \in H_j} P_h(y_{jh}, p^*, y_{jh}^*) y_{jh}$ s.t. $y_j \in Y_j$,

(c) $\Sigma_{i \in I} z_i^* \leq \Sigma_{j \in J} y_j^*$.

Part (a) is the traditional condition saying that households maximize utility as price takers; (c) is the traditional condition of consistency between aggregate plans; (b) says that each firm maximizes profit taking all other prices for goods $h \notin H_j$ as given, and fully taking into account the effect of its quantity decisions y_{jh}, $h \in H_j$, on the prices of the corresponding markets. We may note that the Negishi concept contains the traditional competitive model as a particular case. It suffices to take

$$P_h(y_{jh}, \bar{p}, \bar{y}_{jh}) \equiv \bar{p}_h .$$

Existence conditions are of course a bit more stringent than for a Walrasian equilibrium. In particular the solution in y_j of the profit maximizing program

$$\max \sum_{h \notin H_j} p_h^* y_{jh} + \sum_{h \in H_j} P_h(y_{jh}, p^*, y_{jh}^*) y_{jh}$$

should be convex and u.h.c. in its arguments. This is usually obtained by assuming that the profit function is quasi-concave in the y_{jh}. This assumption is not as demanding as it might seem. Indeed, since the family of perceived demand curves is imagined, they can be chosen so that the various profit functions are actually quasi-concave in the vectors y_{jh} (a natural example is that of isoelastic perceived demand curves).

We may note that we have assumed for notational convenience that the perceived clearing price P_h depends only on y_{jh}. The concept generalizes naturally to the case where P_h depends upon the whole set of $\{y_{jh} \mid h \in H_j\}$ [Negishi (1972)].

6.3. *Objective demand curve: Cournot*

In the preceding section, the prices that would result from a vector y_j were only conjectured by firm j. The idea of an objective demand curve is to replace these conjectures by the prices which would actually obtain, should all firms j take actions y_j, $j = 1, \ldots, n$. This was notably developed in the seminal paper by Gabszewicz and Vial (1972).

In order to make these things more precise, let us simplify the economy in the following way. We assume there are in the economy only non-produced goods (factors of production) sold by households to firms, and produced goods sold by firms to households. In that way no firm sells to another firm. In fact it seems that the concept we shall present below does not generalize readily to a situation where price makers sell to other price makers.

The implicit picture is a two-stage one. In the first stage the firms decide non-cooperatively the production vectors y_j, $j \in J$, in a way we shall describe below. In the second stage a Walrasian equilibrium (possibly) obtains. This Walrasian equilibrium is defined as a price $\hat{p}$ such that

$$\sum_{i=1}^{m} \xi_i\Big(\hat{p}, \sum_{j=1}^{n} \theta_{ij}\hat{p}y_j\Big) = \sum_{j=1}^{n} y_j$$

where the vector functions ξ_i are the Walrasian demand functions of households $i = 1, \ldots, m$. Of course this Walrasian equilibrium, when it exists, will depend on the vectors $y_1, \ldots, y_n$. We shall call *FY* (for feasible y_j's) the set of vectors y_j such that a Walrasian equilibrium exists. We shall further assume that in such a case the equilibrium price is unique, and denote it as $\hat{p}(y_1, \ldots, y_m)$. It is also convenient to rewrite this function from the point of view of firm j as $\hat{p}(y_j, y_{-j})$ where $y_{-j} = \{y_k \mid k \neq j\}$.

Definition 4. A Cournot–Walras equilibrium is defined by a price vector p^*, vectors of production y_j^* and of net trades z_i^* such that

(a) z_i^* maximizes $U_i(\omega_i + z_i, x_i)$ s.t. $p^* z_i + x_i = \bar{x}_i + \Sigma_{j=1}^{n} \theta_{ij} p^* y_j^*$,

(b) y_j^* maximizes $\hat{p}(y_j, y_{-j}^*)y_j$ for all y_j such that $(y_j, y_{-j}^*) \in FY$,

(c) $p^* = \hat{p}(y_1^*, \ldots, y_n^*)$.

Conditions (a) and (c) simply restate that p^* and $x_1^*, \ldots, x_m^*$ form a Walrasian equilibrium relative to the y_j's. Condition (b) says that we have a Nash equilibrium in quantity strategies where each firm maximizes its profit, taking all other firms' quantity strategies as given, and forecasting the price consequences of its choice through the objective price function $\hat{p}$.

We may note that existence in this model poses much more serious problems than in the preceding Negishi concept. Indeed the profit function $\hat{p}(y_j, y_{-j})y_j$ is no more arbitrary, as it is in the subjective demand curve approach, but is fully given by the data of the model. As it turns out, robust examples have been constructed where an equilibrium does not exist and in particular the profit function is not quasi-concave. Roberts and Sonnenschein (1977) have exhibited examples where a pure strategies equilibrium does not exist. Dierker and Grodal (1986) have an example where even a mixed strategies equilibrium does not exist.

Another important point, noted and discussed in Gabszewicz and Vial (1972) and Dierker and Grodal (1986) is that, contrary to the Walrasian case, the equilibrium depends on the normalization rule chosen for prices (we implicitly chose such a normalization by setting the price of the numéraire equal to 1).

We have already noted in Section 4 that in a simple partial equilibrium framework, the Cournot equilibrium is close to a competitive one when each firm is small compared to its market. This issue has been studied in a general equilibrium framework as well, starting with Gabszewicz and Vial (1972), who showed that under suitable conditions the Cournot–Walras equilibrium of a replicated economy would converge towards the competitive equilibrium of the original economy. The point was further studied by Roberts (1980). Hart (1979) analyzed the issue in the case where products could be endogenously selected in a compact set of characteristics, as described in Section 5.2.

6.4. *Objective demand curve with price makers*

We shall now revert to the framework of Section 6.2, where agents are setting prices. A number of concepts of an objective demand curve with price makers have been developed, starting with the pioneering contributions of Marschak and Selten (1974) and Nikaido (1975).[9] We shall describe here a concept developed in Benassy (1988) which takes full advantage of the symmetry between the price-setting and the quantity-setting games.[10] We assume that households as well as firms can set prices. We shall denote by $A = I \cup J$ the set of agents and by H_a the set of prices controlled by agent a. We shall further assume that

$$H_a \cap H_{a'} = \{\emptyset\} , \qquad \bigcup H_a = H .$$

Agents set non-cooperatively prices p_a, $a \in A$ and the equilibrium will be a Nash equilibrium in prices. In order to construct this Nash equilibrium concept, we must be able to forecast the "consequences" of every price vector. The situation is quite symmetric to that encountered in Cournot–Nash equilibrium. There we had to compute the Walrasian equilibrium prices conditional

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[9] See also Laffont and Laroque (1976) and Hart (1985a).

[10] Note that the method described below is also fully applicable to the subjective demand curve approach. See Benassy (1976, 1982, 1990).

on every quantity strategy. Here we must be able to predict quantities demanded, supplied and exchanged conditional on any given price system. Of course the natural theory to use is the theory of fixprice (or non-Walrasian) equilibria which gives an answer to precisely that question. Before constructing the "objective demand curves", let us thus summarize briefly a few concepts [see for example Benassy (1982, 1990) for more details]. At non-Walrasian prices, one should first distinguish between demands and transactions. Call $\tilde{z}_a$ the vector of net demands of agent a, z_a^* the vector of his net transactions. On each market h they are related by

$$z_{ah}^* = \begin{cases} \min\{\tilde{z}_{ah}, \bar{d}_{ah}\} & \tilde{z}_{ah} \geqslant 0\,, \\ \max\{\tilde{z}_{ah}, \bar{s}_{ah}\} & \tilde{z}_{ah} \leqslant 0\,, \end{cases} \tag{18}$$

where $\bar{d}_{ah} \geqslant 0$ and $\bar{s}_{ah} \leqslant 0$ are quantity signals which tell agent a the maximum quantity he can respectively purchase or sell on market h. For a price maker

$$\bar{d}_{ah} = -\sum_{b \neq a} \tilde{z}_{bh} \quad \text{if } a \text{ is a purchaser}\,,$$

$$\bar{s}_{ah} = -\sum_{b \neq a} \tilde{z}_{bh} \quad \text{if } a \text{ is a seller}\,, \tag{19}$$

For the other agents the rationing scheme is usually more complex as there may be many rationed agents on the long side of the market. Transactions and quantity signals will be functions of effective demands,

$$z_a^* = F_a(\tilde{z}_1, \ldots, \tilde{z}_A)\,, \tag{20}$$

$$\bar{d}_a = G_a^d(\tilde{z}_1, \ldots, \tilde{z}_A)\,, \tag{21}$$

$$\bar{s}_a = G_a^s(\tilde{z}_1, \ldots, \tilde{z}_A)\,. \tag{22}$$

Of course in view of relation (18), the functions F_a, G_a^d, G_a^s are not independent. Conversely effective demands are functions of price and quantity signals,

$$\tilde{z}_i = \tilde{\xi}_i(p, \bar{d}_i, \bar{s}_i, \pi)\,, \tag{23}$$

$$\tilde{z}_j = \tilde{\xi}_j(p, \bar{d}_j, \bar{s}_j)\,, \tag{24}$$

where π is the vector of all firm's profits; $\pi_j = -pz_j^*$, $j \in J$.

An equilibrium consists of a set of $\tilde{z}_a$, z_a^*, d_a, $\bar{s}_a$ satisfying equations (20)–(24). As it turns out, for a given rationing scheme it can be proved that an equilibrium exists for all positive prices under fairly standard conditions [Benassy (1982)]. We shall further assume that this equilibrium is unique,[11] and thus write the values of $\tilde{z}_a$, z_a^*, $\bar{d}_a$, $\bar{s}_a$ for price p functionally as $\tilde{Z}_a(p)$, $Z_a^*(p)$, $\bar{D}_a(p)$, $\bar{S}_a(p)$. Now clearly the objective demand and supply curves for agent a are simply represented by the functions $\bar{S}_a(p)$ and $\bar{D}_a(p)$ which represent respectively the maximum quantity of goods h he can respectively sell or purchase as a function of the price vector p. Accordingly the programs determining p_a for $a \in A$ are easily derived. For firm j, p_j is the solution of

$$\text{maximize } py_j = -pz_j \text{ s.t.}$$

$$\begin{cases} y_j \in Y_j \,, \\ \bar{S}_j(p) \leq -y_j \leq \bar{D}_j(p) \,, \end{cases}$$

yielding $p_j = \psi_j(p_{-j})$. For household i, call $\pi_i(p) = -\Sigma_{j \in J} \theta_{ij} p Z_j^*(p)$ his profit income. The vector p_i is the solution of

$$\text{maximize } U_i(\omega_i + z_i, x_i) \text{ s.t.}$$

$$\begin{cases} pz_i + x_i = \bar{x}_i + \pi_i(p) \,, \\ \bar{S}_i(p) \leq z_i \leq \bar{D}_i(p) \,, \end{cases}$$

yielding $p_i = \psi_i(p_{-i})$.

Definition 5. An equilibrium with price makers and objective demand curves is defined as a set of p_j^*, p_i^* such that

(a) $p_i^* \in \psi_i(p_{-i}^*)$, $\forall i$; $p_j^* \in \psi_j(p_{-j}^*)$, $\forall j$;

(b) $\tilde{z}_a$, z_a^*, $\bar{d}_a$, $\bar{s}_a$, $a \in A$, are a fixprice equilibrium relative to p^* and the given rationing schemes, i.e. they are respectively equal to $\tilde{Z}_a(p^*)$, $Z_a^*(p^*)$, $\bar{D}_a(p^*)$, $\bar{S}_a(p^*)$.

Sufficient existence conditions are given in Benassy (1988). As for all models with objective demand curves, these conditions are stronger than the standard assumptions on utility and production functions.

6.5. *Further reading*

The concepts presented in this section can be differentiated along several lines.

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[11] See Schulz (1983) for intuitive sufficient conditions.

One line is that of objective versus subjective demand curves. Subjective demand curves are simpler to use, and do not require more information than that actually observed on markets (i.e. price-quantity pairs). They imbed however a large degree of arbitrariness, notably as far as their slopes are concerned, which leads to a large number of potential equilibria. Objective demand curves on the other hand do not have such arbitrariness, but are very complex objects requiring that each price setter has as much information on the economy as the model maker himself, quite a strong assumption. Some authors have investigated intermediate concepts. Notably Silvestre (1977a) investigates an ingenious equilibrium concept where each firm knows locally the true slope of its demand curve. Gary-Bobo (1989) shows that under suitable assumptions these equilibria are the same as objective demand curves equilibria.

Another line of differentiation is that of quantity-setting models (Cournot) versus price-setting models (Chamberlin), which we have already seen in Section 4. An interesting attempt at reconciliation is the idea of rational conjectures put forward by Hahn (1977, 1978), according to which each agent would (rationally) conjecture the price-quantity responses of other agents. A fully satisfactory concept does not seem to exist yet, however.

Throughout this section we have assumed that firms maximize profits. Though this is a most traditional assumption, it is clear that the adequate criterion should rather be some kind of weighted average of shareholders' utilities. On this issue see for example Dierker and Grodal (1986), Gabszewicz and Vial (1972) and Mas-Colell (1984).

Another most interesting development in the field is the relation with increasing returns, which was forcefully put forward by Sraffa (1926). We have already mentioned general equilibrium Cournotian models with increasing returns. A number of contributions [Arrow and Hahn (1971), Silvestre (1977b, 1978)] have included increasing returns in general equilibrium models with price making agents. See also the 1988 special issue of the Journal of Mathematical Economics on the subject.

Finally, following the monopolistic competition tradition, we have restricted ourselves to models with one sided price (or quantity) setting. Strategic market games with two sided price and (or) quantity setting have been constructed, following notably Shubik (1973) and Shapley and Shubik (1977). In Cournotian-type market games, the equilibrium price system generally converges towards the Walrasian one as the number of agents on each side of the markets becomes large [see for example Dubey and Shubik (1978), Postlewaite and Schmeidler (1978), Dubey, Mas-Colell and Shubik (1980), Mas-Colell (1982)]. In Bertrand–Edgeworth type games a Walrasian outcome can be obtained with only a few agents one each side of every market [see for example Dubey (1982), Benassy (1986a)].

7. Monopolistic competition and macroeconomic issues

The use of general equilibrium models such as the ones presented in the preceding section allows us to construct rigorous "micro-macro" models of monopolistic competition. Within such a framework one can study macroeconomic issues, such as the existence of unemployment or the effectiveness of government policies, as well as more traditional microeconomic issues, such as the efficiency of equilibrium. We shall present in this section a complete model of this type, which will show that with respect to the above issues, monopolistic competition leads to results somehow "intermediate" between Walrasian models and traditional Keynesian fixprice models.

7.1. The model

We shall consider a monetary economy with three types of goods: fiat money, which is the numéraire, medium of exchange and a store of value; different types of labor indexed by $i = 1, \ldots, m$; and consumption goods indexed by $j = 1, \ldots, n$. There are three types of agents: households indexed by $i = 1, \ldots, m$; firms indexed by $j = 1, \ldots, n$; and government. Consumer i is the only one to be endowed with labor of type i, firm j is the only one to produce good j. We shall call w_i the money wage for type i labor, p_j the price of good j, w and p the corresponding vectors:

$$p = \{p_j \mid j = 1, \ldots, n\}, \qquad w = \{w_i \mid i = 1, \ldots, m\}.$$

Firm j produces a quantity of output y_j according to a production function

$$y_j = F_j(l_j), \qquad l_j = \{l_{ij} | i = 1, \ldots, m\}$$

where l_{ij} is the quantity of labor i used by firm j (and thus purchased from household i). We shall assume F_j strictly concave in its arguments. Firm j maximizes its profits π_j:

$$\pi_j = p_j y_j - w l_j = p_j y_j - \sum_{i=1}^{m} w_i l_{ij}.$$

Household i has initial endowments l_{i0} of type i labor and $\bar{m}_i$ of money. He consumes a vector $c_i = \{c_{ij} \mid j = 1, \ldots, n\}$ and works a quantity of labor l_i:

$$l_i = \sum_{j=1}^{n} l_{ij} \leq l_{i0}. \tag{25}$$

Household *i's* budget constraint is

$$pc_i + m_i = w_i l_i + \mu \bar{m}_i + \sum_{j=1}^{n} \theta_{ij} \pi_j$$

where m_i is the final quantity of money and θ_{ij} the share of firm j owned by household i. The factor μ is a government policy instrument whereby government can increase proportionately all money holdings by the same factor μ. This particular (but popular) policy has been chosen because it is known to be "neutral" in Walrasian equilibrium,[12] which will allow an easy comparison.

Household i maximizes a utility function of the form

$$U_i(c_i, l_i, m_i/\mu)$$

which is assumed to be strictly quasi-concave in c_i, l_i and m_i/μ and separable in its arguments. We shall assume that the disutility of labor becomes so high near l_{io} that constraint (25) is never binding. The argument m_i/μ represents the indirect utility of money, which should be homogeneous of degree zero in money and expected prices. Thus the implicit idea behind our assumption is that, other things being equal, future prices are expected to move proportionately to μ. As we shall see in Section 7.5, this proportionality of prices to μ is consistent with the working of the model.[13]

This model has a particular market structure where each good (labor or consumer good) is sold by a single agent to many other agents; labor of type i is sold by household i to all firms $j = 1, \ldots, n$. Output j is sold by firm j to all households $i = 1, \ldots, m$. We shall assume that the price is decided upon by the single seller. Firm j sets price p_j, household i sets wage w_i, taking all other prices and wages as given. We shall assume that they do so using objective demand curves, as described in Section 6.4. The equilibrium is thus a Nash equilibrium in prices and wages, conditionally on these objective demand curves.

7.2. Objective demand curves

Each seller sells only one good, and, as we shall see below, sets his price high enough so as to be willing to satisfy all demand for that good. In equilibrium each agent will thus be constrained only on his sales, and we shall have

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[12] See for example Grandmont (1983) for a thorough discussion of this issue in Walrasian models.

[13] For explicitly dynamic models with perfect foresight which exhibit this proportionality property, see for example Benassy (1989b, 1991).

somehow a situation of "general excess supply". We shall now compute the objective demand curves in this zone of general excess supply. These objective demand curves will be functions of the vectors p and w, and of the policy parameter μ. So for given p, w and μ we must find out which demands for goods and labor types will arise once all feedback effects have been taken into account. As indicated in Section 6.4 this boils down to finding total demand for goods i and j at a fixprice equilibrium corresponding to p, w and μ.

For given (p, w, μ) firm j is constrained on its sales of output j. It thus solves the following program in l_j:

$$\text{maximize } p_j y_j - w l_j \text{ s.t. } y_j \leqslant F_j(l_j)$$

where y_j is a binding constraint exogenous to the firm. The solution is a set of labor demands $L_{ij}(y_j, w)$ and a cost function $\chi_j(y_j, w)$.

Similarly consider a household i which solves the following program:

$$\text{maximize } U_i(c_i, l_i, m_i/\mu) \text{ s.t.}$$

$$pc_i + m_i = \mu \bar{m}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij} \pi_j$$

where p, w, μ, l_i and the profits π_j are given. The solution is a set of consumption demands,

$$C_{ij}\left(\mu \bar{m}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij} \pi_j, p, \mu \right).$$

Consider now the following mapping:

$$y_j \rightarrow \sum_{i=1}^{m} C_{ij}\left(\mu \bar{m}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij} \pi_j, p, \mu \right),$$

$$l_i \rightarrow \sum_{j=1}^{n} L_{ij}(y_j, w),$$

$$\pi_j \rightarrow p_j y_j - \chi_j(y_j, w).$$

Assuming a unique fixed point, this yields functions $Y_j(p, w, \mu)$, $L_i(p, w, \mu)$ and $\pi_j(p, w, \mu)$ which represent respectively the objective demand for good j, for labor i and the associated profits of firm j.

Secondly let us note that the functions L_{ij} are homogeneous of degree 0 in w, the functions C_{ij} are homogeneous of degree 0 in μ, w_i, p and π_j, and profits are homogeneous of degree 1 in p and w. From that we deduce Y_j and L_i are homogeneous of degree 0, and π_j homogeneous of degree 1, in the arguments p, w and μ.

7.3. *Equilibrium: definition and characterization*

Following Section 6.4 we shall describe the equilibrium as a Nash-equilibrium in prices and wages, conditional on the objective demand curves. The quantities are those corresponding to a fixprice equilibrium associated with the prices and wages. We shall now derive the optimal price and wage responses of the agents.

Consider first firm j; it will solve the following profit maximization program A_j in p_j, y_j and l_j:

$$\text{maximize } p_j y_j - w l_j \text{ s.t.}$$
$$\begin{cases} y_j \leqslant F_j(l_j)\,, \\ y_j \leqslant Y_j(p, w, \mu)\,. \end{cases} \tag{A_j}$$

We assume this program has a unique solution, which thus yields optimal price p_j as a function of the other prices and wages,

$$p_j = \psi_j(p_{-j}, w, \mu)$$

where $p_{-j} = \{p_k \mid k \neq j\}$. In order to characterize the equilibrium quantities simply, we shall also need to describe the optimal production plan of the firm (y_j, l_j) as a function of the same variables, so that we shall write this optimal plan functionally as

$$(y_j, l_j) = \varphi_j(p_{-j}, w, \mu)\,.$$

Consider now household i. It chooses the wage w_i, labor sales l_i and consumption vector c_i so as to maximize utility according to the following program A_i in w_i, l_i and c_i:

$$\text{maximize } U_i(c_i, l_i, m_i/\mu) \text{ s.t.}$$
$$\begin{cases} pc_i + m_i = \mu\bar{m}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij}\pi_j(p, w, \mu)\,, \\ l_i \leqslant L_i(p, w, \mu)\,, \end{cases} \tag{A_i}$$

which yields the functions

$$w_i = \psi_i(w_{-i}, p, \mu)\,,$$
$$(l_i, c_i) = \varphi_i(w_{-i}, p, \mu)\,,$$

where $w_{-i} = \{w_k \mid k \neq i)$. We can now define our equilibrium with monopolistic competition as a Nash equilibrium as follows:

(a) $w_i^* = \psi_i(w_{-i}^*, p^*, \mu)$, $\forall i$;

(b) $p_j^* = \psi_j(p_{-j}^*, w^*, \mu)$, $\forall j$,

(c) $(l_i^*, c_i^*) = \varphi_i(w_{-i}^*, p^*, \mu)$, $\forall i$;

(d) $(y_j^*, l_j^*) = \varphi_j(p_{-j}^*, w^*, \mu)$, $\forall j$.

The first two sets of equalities express the Nash property in prices and wages, and determine p^* and w^*. The last two sets of equations give the optimal production, work and consumption plans of the agents, which correspond to their transactions on all markets. Note that because of the definition of the objective demand curves, which is based on a fixprice equilibrium notion, the consistency of transactions is automatically ensured, so that we do not need to add the traditional equations stating the equality between purchases and sales on every market.

Of course the equilibrium will change as μ changes. In what follows we shall assume that to a given government policy is associated a unique equilibrium. In order to study its various properties, we shall now characterize it by deriving a number of relevant partial derivatives.

Consider first firm j and recall the program yielding its optimal actions:

$$\text{maximize } p_j y_j - w l_j \text{ s.t.}$$
$$\begin{cases} y_j = F_j(l_j)\,, \\ y_j \leq Y_j(p, w, \mu)\,. \end{cases} \tag{A_j}$$

Let us assume an interior solution, so that in particular all components of l_j are strictly positive. Then the Kuhn–Tucker conditions associated with this program yield immediately

$$\frac{\partial F_j}{\partial l_{ij}} = \frac{w_i}{p_j}\,\frac{1}{1-(1/\varepsilon_j)} \tag{26}$$

where $\varepsilon_j = -(p_j/y_j)\,\partial Y_j/\partial p_j$ is the absolute value of the own price elasticity of objective demand. At an equilibrium, ε_j is greater than 1. These conditions can also be rewritten, using the cost function $\chi_j(y_j, w)$ of firm j:

$$\frac{\partial \chi_j}{\partial y_j} = p_j\left(1 - \frac{1}{\varepsilon_j}\right) \tag{27}$$

which is the traditional "marginal cost equals marginal revenue" equations. Let us turn now to the optimal program of household i:

$$\text{maximize } U_i(c_i, l_i, m_i/\mu) \text{ s.t.}$$
$$\begin{cases} pc_i + m_i = \mu \bar{m}_i + w_i l_i + \sum_{j=1}^{n} \theta_{ij}\pi_j(p, w, \mu)\,, \\ l_i \leq L_i(p, w, \mu)\,. \end{cases} \tag{A_i}$$

We shall assume an interior solution so that in particular all consumptions are positive. Call λ_i the "marginal utility of wealth", i.e. the Kuhn–Tucker multiplier of the budget constraint. We obtain the following conditions:[14]

$$\frac{\partial U_i}{\partial m_i} = \lambda_i \,, \qquad \frac{\partial U_i}{\partial c_{ij}} = \lambda_i p_j \,, \tag{28}$$

$$\frac{\partial U_i}{\partial l_i} = -\lambda_i w_i \left(1 - \frac{1}{\varepsilon_i}\right), \tag{29}$$

where $\varepsilon_i = -(w_i/l_i)\,\partial L_i/\partial w_i$. Again, at equilibrium $\varepsilon_i > 1$. With the help of these differential characterizations, we can now describe some salient properties of our equilibrium.

7.4. Underemployment, underproduction and inefficiency

We shall now show that, even though prices are fully flexible and rationally decided upon by agents, the equilibrium allocation has properties which strongly differentiate it from those of a Walrasian equilibrium.

First we see that at equilibrium there is both underemployment and underproduction. Indeed equation (27) shows that at the going price and wage, firm j would be happy to produce and sell more if the demand was forthcoming, thus displaying underproduction. Symmetrically, equation (29) shows that household i would like to sell more of its labor if the demand was present, thus displaying underemployment.

We shall further show that employment and production throughout the economy are inefficiently low in the following strong sense: it is possible to find increases in production and employment which would increase all firms' profits and all households' utilities at the equilibrium prices and wages.

Consider indeed at the given price and wage system p^* and w^*, some small arbitrary increases $\mathrm{d}l_{ij} > 0$. These yield extra amounts of employment and production:

$$\mathrm{d}l_i = \sum_{j=1}^{n} \mathrm{d}l_{ij} > 0$$

$$\mathrm{d}y_j = \sum_{i=1}^{m} \frac{\partial F_j}{\partial l_{ij}} \mathrm{d}l_{ij} > 0$$

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[14] We actually assume for simplicity that the influence of w_i on household i's profit income is negligible, which will be the case if there are many households, and each owns negligible shares of each firm.

Consider first the profit variation for firm j:

$$\mathrm{d}\pi_j = p_j\,\mathrm{d}y_j - \sum_{i=1}^{m} w_i\,\mathrm{d}l_{ij}$$

In view of equations (26), this is easily found equal to

$$\mathrm{d}\pi_j = \frac{p_j\,\mathrm{d}y_j}{\varepsilon_j} > 0 \tag{30}$$

Now we shall assume that the extra production in the economy is redistributed to households in such a way that the values of the extra consumptions for each household add up to the value of his extra labor and profit income, which is written for household i,

$$\sum_{j=1}^{n} p_j\,\mathrm{d}c_{ij} = w_i\,\mathrm{d}l_i + \sum_{j=1}^{n} \theta_{ij}\,\mathrm{d}\pi_j\,. \tag{31}$$

The increment in utility is, since m_i and μ do not change,

$$\mathrm{d}U_i = \sum_{j=1}^{n} \frac{\partial U_i}{\partial c_{ij}}\,\mathrm{d}c_{ij} + \frac{\partial U_i}{\partial l_i}\,\mathrm{d}l_i$$

which, using equations (23)–(26) becomes

$$\mathrm{d}U_i = \lambda_i\left[\frac{w_i\,\mathrm{d}l_i}{\varepsilon_i} + \sum_{j=1}^{n} \frac{\theta_{ij}p_j\,\mathrm{d}y_j}{\varepsilon_j}\right] > 0 \tag{32}$$

Equations (30) and (32) clearly show that the incremental employment and production will increase all agents' utilities or profits. We may note that this inefficiency result is quite stronger than Pareto inefficiency since we have constrained the incremental trades to be consistent with the equilibrium price wage system, whereas such a constraint is not required to obtain Pareto inefficiency. We should also note that these inefficiencies are quite similar to these observed in "Keynesian type" general excess supply states [see for example Benassy (1977, 1982)]. We may also finally note that the "efficiency losses", as described by equations (30) and (32) are higher, the higher the Lerner indices $1/\varepsilon_i$ and $1/\varepsilon_j$ on the various markets.

7.5. *Neutrality of monetary policy*

We have seen in the preceding subsections that our imperfect competition equilibrium displayed some inefficiency properties very akin to traditional

Keynesian excess supply states. We shall now investigate a traditional "Keynesian" policy to cure such inefficiencies, a monetary policy, and we shall now see that, in spite of its "Keynesian" characteristics, the system described above reacts to monetary policy in a more "Walrasian" than "Keynesian" manner. Namely, monetary policies taking the form of proportional increases in initial money holdings (i.e. $\mu > 1$) are ineffective, or "neutral", just as in Walrasian models. As a response to such a policy, production, employment and utilities do not change. Prices, wages and profits are multiplied by μ.

The proof of that result is actually quite trivial in view of the homogeneity properties of our system. Let us look indeed at the programs A_j and A_i yielding the optimal actions of agents; from program A_j it is clear that

$$\psi_j(\mu p_{-j}, \mu w, \mu) = \mu \psi_j(p_{-j}, w, 1) ,$$
$$\varphi_j(\mu p_{-j}, \mu w, \mu) = \varphi_j(p_{-j}, w, 1) .$$

Similarly program A_i yields

$$\psi_i(\mu w_{-i}, \mu p, \mu) = \mu \psi_i(w_{-i}, p, 1) ,$$
$$\varphi_i(\mu w_{-i}, \mu p, \mu) = \varphi_i(w_{-i}, p, 1) .$$

In view of these homogeneity properties, y_j^*, l_j^*, l_i^*, c_i^* are homogeneous of degree 0 in μ, whereas p^* and w^* are homogeneous of degree 1 in μ.

7.6. *Further reading*

The model presented here, developed in Benassy (1987, 1990), is a model with explicit price makers and objective demand curves. Similar models with unemployment were presented in Benassy (1977) and Negishi (1977) for subjective demand curves with price makers, and in Hart (1982) for an objective "Cournotian" demand curve with quantity setting agents.

Other macroeconomic models with imperfect competition are found in Benassy (1982, 1989b, 1991), D'Aspremont, Dos Santos and Gérard-Varet (1989, 1990), Dehez (1985), Dixon (1987c), Jacobsen and Schultz (1990), Negishi (1979), Silvestre (1988), Sneessens (1987), Snower (1983), Svensson (1986) and Weitzman (1982, 1985).

8. Conclusions

Traditional microeconomic theory, whether Marshallian or Walrasian, was clearly in need of an explicit theory of price making by agents internal to the system. Monopolistic competition is obviously an interesting and fruitful attempt in this direction. It allows us to deal with problems which competitive theory could not cope with, such as increasing returns. It yields new insights in fields such as macroeconomics or international trade [on the latter, which we did not survey at all, see for example the recent expositions by Helpman (1989), Helpman and Krugman (1985), Krugman (1989)].

In view of the size of the field, the overview of this chapter could only be a partial one, and the reader will find complementary insights in other surveys, such as for example Archibald (1987), Archibald, Eaton and Lipsey (1985), Eaton and Lipsey (1989), Gabszewicz and Thisse (1986), Hart (1985a), Ireland (1987), Lancaster (1979), Mas-Colell (1982), Negishi (1987), Novshek and Sonnenschein (1986, 1987), Shubik (1959, 1985), Shubik and Levitan (1980) and Stiglitz (1986).

As indicated in the introduction, in order to have a fairly homogeneous presentation, we restricted our exposition to the traditional vision of a monopolistic competition equilibrium as a one period Nash equilibrium with perfect information. Already in this apparently simple framework we encountered a number of important conceptual problems. In particular it appears that a synthesis between the lines of research initiated by Cournot, Bertrand, Edgeworth, Hotelling and Chamberlin remains to be done.

In spite of the potential difficulties, it is nonetheless clear that research on the monopolistic competition paradigm should be pursued in a number of directions. Notably more complex strategic interactions should be considered, such as those found in the theory of oligopoly or industrial organization [see for example Friedman (1982), Shapiro (1989), Tirole (1988)], imbedding of course the time dimension of the games actually played. Also, in view of the extremely high amount of information implicit in the games we described, as compared with the competitive price taking paradigm, the study of imperfect information seems a natural extension of the theory of monopolistic competition [see for example Stiglitz (1989)]. All this should be the subject of fruitful further research.

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