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Optimal indexation and the cyclical behavior of prices

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Abstract

We construct in this article a dynamic model with technological and monetary shocks. We show that, whereas prices may be procyclical without indexation, they are countercyclical under optimal indexation.

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1. Introduction

When discussing whether prices are pro or countercyclical, one usually finds in the literature two different traditions, and answers. On the one hand, authors in the real business cycles tradition particularly emphasize productivity shocks, and show that these will lead to countercyclical prices. On the other hand, authors in the Keynesian tradition believe that monetary shocks will lead to procyclical prices.

From an empirical point of view, it seems that the relation between prices and output has evolved in time. For example Cooley and Ohanian (1991) and Smith (1992) view prices as procyclical in the interwar period, but countercyclical in the postwar period.

We will show in this article that a possible explanation for this transition to countercyclical prices may be the growing importance of indexation clauses in wage contracts. For that purpose we shall consider a simple dynamic model with technological and monetary shocks, derive the optimal rate of indexation[1],

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[1] The literature on optimal macroeconomic indexation begins with Gray (1976). An article deriving optimal indexation in a fully maximizing model is Hénin and Zylberberg (1986).

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and show that under such indexation prices are countercyclical, whereas they can be procyclical without indexation.

2. The model

We consider a dynamic monetary model. Firms have a simple technology:

$$Y_t = Z_t L_t^\alpha \tag{1}$$

where Y_t is output, L_t labor and Z_t a technology shock. Profits Π_t are distributed to the consumers. The consumers have the following utility:

$$\Sigma \beta^t (\log C_t - \phi L_t^\gamma) \tag{2}$$

The consumer starts period t with financial assets A_{t-1} coming from period $t-1$. We shall assume that there is a zero net bond balance in the economy at all times. At the beginning of period t the consumer receives a lump sum monetary transfer T_t. He is subject to a "cash in advance" constraint:

$$P_t C_t \leq A_{t-1} + T_t = M_t \tag{3}$$

Then the consumer receives wage income $W_t L_t$ and profits Π_t, so that his wealth A_t at the end of period t is given by:

$$A_t = M_t - P_t C_t + W_t L_t + \Pi_t \tag{4}$$

Finally we assume that the goods market clears, but that the wage is given by the following indexing formula (in logarithms)[2]:

$$w_t = \omega_t + v p_t \qquad 0 \leq v \leq 1 \tag{5}$$

where v is the degree of indexation and ω_t is a preset "base value" for the wage. Both parameters will be optimally determined below.

We assume that the technology and monetary shocks z_t and m_t are normal variables, and denote:

$$\varepsilon_{zt} = z_t - E_{t-1} z_t \quad \varepsilon_{mt} = m_t - E_{t-1} m_t \quad \sigma_z^2 = \mathrm{var}(\varepsilon_{zt}) \quad \sigma_m^2 = \mathrm{var}(\varepsilon_{mt}) \tag{6}$$

3. Equilibrium

It is shown in Appendix A that, provided that $1/M_t > \beta E_t(1/M_{t+1})$, which we will assume in what follows, the cash in advance constraint (3) is satisfied with equality so that in logarithms:

$$m_t = p_t + c_t = p_t + y_t \tag{7}$$

[2] Lowercase letters denote the logarithms of the corresponding uppercase letters.

Now since the production function (1) is isoelastic and the goods market clears:

$$y_t = z_t + \alpha l_t \tag{8}$$

$$w_t + l_t = p_t + y_t + \log\alpha \tag{9}$$

Combining these three equations and the indexation formula (5) yields the value of equilibrium employment:

$$l_t = \frac{vz_t + (1-v)m_t - \omega_t + \log\alpha}{1-\alpha v} \tag{10}$$

4. Optimal indexation

The values of v and ω_t are chosen at the beginning of period t, before the shocks are known, so as to maximize the ex-ante expected utility of consumers, which is equal to:

$$E_{t-1}(y_t) - \phi E_{t-1}L_t^\gamma = E_{t-1}z_t + \alpha E_{t-1}l_t - \phi E_{t-1}L_t^\gamma \tag{11}$$

Since L_t is lognormal, we have the usual formula:

$$E_{t-1}L_t^\gamma = \exp\left[\gamma E_{t-1}(l_t) + \frac{\gamma^2 V_{t-1}(l_t)}{2}\right] \tag{12}$$

where, from Eq. (10), the conditional mean $E_{t-1}(l_t)$ is equal to:

$$E_{t-1}(l_t) = \frac{vE_{t-1}z_t + (1-v)E_{t-1}m_t - \omega_t + \log\alpha}{1-\alpha v} \tag{13}$$

and the variance $V_{t-1}(l_t)$:

$$V_{t-1}(l_t) = \frac{v^2\sigma_z^2 + (1-v)^2\sigma_m^2}{(1-\alpha v)^2} \tag{14}$$

Combining (11) and (12), we find that the expected utility of the consumer (11), as of the beginning of period t, becomes:

$$E_{t-1}z_t + \alpha E_{t-1}l_t - \phi\exp\left[\gamma E_{t-1}(l_t) + \frac{\gamma^2 V_{t-1}(l_t)}{2}\right] \tag{15}$$

We want to maximize this expression with respect to v and ω_t. We can actually take $E_{t-1}(l_t)$ and $V_{t-1}(l_t)$ as intermediary maximizing variables. The first optimality condition corresponds to minimization of $V_{t-1}(l_t)$, which yields, using (14):

$$v = \frac{(1-\alpha)\sigma_m^2}{\sigma_z^2 + (1-\alpha)\sigma_m^2} \tag{16}$$

Formula (16) gives the optimal rate of indexation. Now, maximization of (15) with respect to $E_{t-1}(l_t)$ yields:

$$E_{t-1}(l_t) = \frac{1}{\gamma}\log\left(\frac{\alpha}{\phi\gamma}\right) - \frac{\gamma V_{t-1}(l_t)}{2} \tag{17}$$

which, using (13), translates into the following value for ω_t:

$$\omega_t = vE_{t-1}z_t + (1-v)E_{t-1}m_t + \log\alpha + (1-\alpha v)\left[\frac{\gamma V_{t-1}(l_t)}{2} - \frac{1}{\gamma}\log\left(\frac{\alpha}{\phi\gamma}\right)\right] \tag{18}$$

5. The countercyclicality of prices

Combining Eqs. (7), (8), (10) and (18), we find that the values of output and prices are (omitting irrelevant constants):

$$y_t = E_{t-1}z_t + \frac{\varepsilon_{zt} + \alpha(1-v)\varepsilon_{mt}}{1-\alpha v} \tag{19}$$

$$p_t = E_{t-1}m_t - E_{t-1}z_t + \frac{(1-\alpha)\varepsilon_{mt} - \varepsilon_{zt}}{1-\alpha v} \tag{20}$$

We see that in order to compute the price–output covariance we need to specify more the technology process. We will make the usual assumption:

$$z_t = \frac{\varepsilon_{zt}}{1-\rho L} \qquad 0 \leq \rho < 1 \tag{21}$$

So the covariance of output and prices is:

$$\mathrm{cov}(y_t, p_t) = \frac{\alpha(1-\alpha)(1-v)\sigma_m^2 - \sigma_z^2}{(1-\alpha v)^2} - \frac{\rho^2\sigma_z^2}{1-\rho^2} \tag{22}$$

We first see that in the absence of indexation ($v = 0$), prices can be procyclical provided demand shocks are sufficiently volatile, more precisely provided that:

$$\alpha(1-\alpha)\sigma_m^2 > \frac{\sigma_z^2}{1-\rho^2} \tag{23}$$

Now we will see that under optimal indexation the situation changes completely. Indeed, inserting into (22) the value of the optimal degree of indexation (16), we find:

$$\mathrm{cov}(y_t, p_t) = -\sigma_z^2\frac{\sigma_z^2 + (1-\alpha)\sigma_m^2}{\sigma_z^2 + (1-\alpha)^2\sigma_m^2} - \frac{\rho^2\sigma_z^2}{1-\rho^2} \tag{24}$$

We see that this covariance is always negative, whatever the respective values of σ_z^2 and σ_m^2, so that prices are always countercyclical.

We may note that the result of countercyclicality of prices is actually not restricted to optimal indexation. It is easy to compute, using formula (22), that a sufficient condition for prices to be countercyclical is that:

$$1 - \nu < \frac{\sigma_z^2}{\alpha(1-\alpha)\sigma_m^2}\left[1 + \frac{\rho^2(1-\alpha)^2}{1-\rho^2}\right] \tag{25}$$

i.e. that the degree of indexation be large enough. We may note that the optimal degree of indexation is always in this range.

6. Conclusions

So, to summarize, we have found that in our model: (a) in the absence of indexation, prices can be procyclical, provided demand shocks are volatile enough (formula 23); (b) if the degree of indexation is large enough prices are countercyclical (formula 25); (c) optimal indexation leads to countercyclical prices whatever the relative size of the shocks (formula 24).

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Appendix A

Since $M_t = A_{t-1} + T_t$ the consumers' budget constraint (4) can be rewritten:

$$A_t = A_{t-1} + T_t - P_tC_t + W_tL_t + \Pi_t \tag{26}$$

The consumer maximizes his utility (2) subject to this constraint and the cash in advance constraint (3). The Lagrangean is written:

$$\sum \beta^t[\log C_t - \phi L_t^\gamma + \mu_t(A_{t-1} + T_t - P_tC_t)] + \sum \beta^t \lambda_t(A_{t-1} + T_t - P_tC_t + W_tL_t + \Pi_t - A_t) \tag{27}$$

The first order conditions with respect to C_t and A_t are:

$$\frac{1}{C_t} = (\lambda_t + \mu_t)P_t \tag{28}$$

$$\lambda_t = \beta E_t(\lambda_{t+1} + \mu_{t+1}) \tag{29}$$

Let us assume to start with that the cash in advance constraint is binding, so that $P_tC_t = M_t$. Then from (28):

$$\lambda_t + \mu_t = \frac{1}{M_t} \tag{30}$$

Inserting this into (29) we obtain:

$$\lambda_t = \beta E_t\left(\frac{1}{M_{t+1}}\right) \tag{31}$$

Now the cash in advance constraint will be actually binding if $\mu_t > 0$, i.e. if, combining (30) and (31):

$$\frac{1}{M_t} > \beta E_t\left(\frac{1}{M_{t+1}}\right) \tag{32}$$

which we will always assume.

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